Math 166: Honors Calculus II Name:________________________

Final Exam *May 8, 1996*

There are 12 problems worth a total of 170 pts.

1. (10 pts) State and prove the First Fundamental Theorem of Calculus.

2. (10 pts)

a) Compute $\int_{-1}^{0} \frac{x}{\sqrt{x+2}}\,dx$

b) Integrate $\int e^{-x}\cos(x)\,dx$

3. (20 pts) Give complete definitions of the following functions, including domains and ranges.

   a) $\log(x)$

   b) $\log_a(x)$ for $a > 0$

   c) $\exp(x)$

   d) a^x for $a > 0$

4. (10 pts)

a) Derive $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$.

b) Let $f(x) = \frac{1}{1+e^x}$. Determine the interval(s) on which $f(x)$ has an inverse and derive a formula for $f^{-1}(x)$. Be sure to give the domain of $f^{-1}(x)$.

5. (10 pts) Compute the following derivatives.

a) $\dfrac{d}{dx}\displaystyle\int_{\log(x)}^{x^2} \cos(e^x)\,dx.$

b) $\dfrac{d}{dx}\log(x)^{\log(x)}$

6. (15 pts)

a) Compute the partial fraction decomposition of $\dfrac{-2+x+9x^2+4x^3}{(x^2+x)(x^2-1)}$

b) Use an appropriate substitution to transform the integral

$$\int \frac{1}{1+\tan(\theta)}\,d\theta$$

into an integral of a rational function (do not evaluate the integral).

c) Integrate $\displaystyle\int \frac{1}{\sqrt{2x^2+4x+1}}\,dx.$

7. (15 pts)

a) State Taylor's Formula with Lagrange Remainder.

b) Find $T_{3k+2}\left(\dfrac{x^2}{x^3+2}\right)$.

c) Find $T_4(\sin(\log(1+x)))$.

8. (20 pts) Compute the following limits.

a) $\lim_{x \to 0} \dfrac{\sin(x^4)}{x^2(e^x - 1)^2}$

b) $\lim_{x \to 0} \dfrac{3^x - 5^x}{x}$

c) $\lim_{x \to 0} \dfrac{1}{x^2} - \dfrac{1}{e^{x^2} - 1}$

d) $\lim_{n \to \infty} \dfrac{\sqrt[n]{n^2 + (-1)^n}}{\sqrt{n^4 + n}}$

9. (10 pts) State and prove the Monotone Convergence Theorem.

10. (20 pts) Compute the value of the following infinite series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}$

b) $\displaystyle\sum_{n=5} \frac{2+(-1)^n}{3^n}$

c) $\displaystyle\sum_{n=1}^{\infty} 2nx^{2n+1}$ for $|x| < 1$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$

11. (20 pts) Test whether the following series converge absolutely, conditionally, or diverge. Justify your answers.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\log(n)^s}$ for $0 < s < 1$.

b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{1/n}$

c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\log(n+1)}{\sqrt{n(n+1)}}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(-2n)^n}$

12. (10 pts) Determine the interval of convergence for the following power series.

a) $\sum_{n=0}^{\infty}(1+(-3)^n)x^n$

b) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+2)3^n}$