University of Notre Dame
Math. 166, Final Examination (May 8,1997)

Instructor: Hong-Ming Yin, **Duration: 2 hours.**

[30] 1. The following statements may be true or false. If it is true, give a short proof, if not, give a counterexample or give a true statement.
(a) Let $f(x)$ and $g(x)$ be defined on $[0,\infty)$ and

$$\lim_{x\to+\infty} f(x) = +\infty, \lim_{x\to+\infty} g(x) = +\infty;$$

Then

$$\lim_{x\to+\infty} [f(x) - g(x)] = 0.$$

(b) Let z be a complex number and i be the imaginery unit. Then

$$e^{z+2n\pi i} = e^z \qquad \text{for all integer } n.$$

(c) Let $\{a_n\}$ be a non-negative and monotone decreasing sequence. Moreover,

$$\lim_{n\to\infty} a_n = 0$$

then the series $\sum_{n=1}^{\infty} \frac{a_n}{\sqrt{n}}$ converges.

(d) For all $x \in (-1,\infty)$,

$$log(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} x^n.$$

(e) The function sequence $\{f_n(x)\} = \{log(1+x^n)\}$ converges uniformly on $[0,1]$.

(f) The following identity holds for all x:

$$\arcsin x + \arccos x = 1.$$

[20] 2.(a) Find the values of a and b such that

$$\int_2^{+\infty} \left[\frac{2x^2 + bx + a}{x(2x+a)} - 1\right] dx = 1.$$

(b) Prove the improper integral

$$\int_1^{\infty} \frac{x^s}{e^x} dx$$

is convergent for any fixed $s \in (-\infty, +\infty)$.

[20] 3. For each of the following power series, determine the set of all real x for which the series converges.

$$(a) \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{log(1+n)}} x^n;$$

$$(b) \qquad \sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$$

[20] 4. Find the Taylor's series centered at 0 of the following functions and indicate the radius of convergence for the series:

(a) $f(x) = sin2x$

(b) $g(x) = \frac{1}{2-x-x^2}$

[10] 5. Let $f(x) = e^{-\frac{1}{x^2}}$ if $x \neq 0$ and $f(0) = 0$.
(a) Prove that for every integer $m > 0$,

$$\lim_{x \to 0} \frac{f(x)}{x^m} = 0;$$

(b) Prove $f^{(n)}(0) = 0$ for all $n \geq 1$.