

**Exam I** *Feb. 18, 1999*

There are 6 questions, each with several parts, worth a total of 110 points. Be sure to show all your work and justify all steps.

1. (30 pts) Define each of the following functions precisely, including their domains.

a) The natural logarithm function,  $\log(x)$ .

b) The natural exponential function,  $\exp(x)$ .

c)  $a^x$  for  $a > 0$ .

d)  $\tanh(x)$

e)  $\arccos(x)$

f)  $T_n f(x; a)$

2. (10 pts) State each of the following theorems precisely.

a) The First Fundamental Theorem of Calculus.

b) The Theorem on Derivatives of Inverse Functions.

3. (15 pts)

a) Compute  $\frac{d}{dx} \int_{-\log(x)}^{\log(x)} \sin(e^t) dt$

b) Compute  $\frac{d}{dx} \cos(x)^{\sin(x)}$ .

c) Prove  $\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$ .

4. (30 pts) Compute the following integrals.

a)  $\int \frac{x-1}{\sqrt{x-2}} dx$

b)  $\int x^2 e^x dx$

c)  $\int \log x dx$

$$\text{d) } \int \frac{1}{(x^2 + 1)^2} dx$$

$$\text{e) } \int \frac{1}{\sqrt{1 + x - x^2}} dx$$

$$\text{f) } \int \frac{1}{x^2 - 5x + 7} dx$$

5. (15 pts)

a) Give the partial fraction decomposition of  $\frac{4x^3 - x^2 - 4x + 5}{(x - 1)^2(x^2 + 1)}$   
(Solve for the constants).

b) Use a trigonometric substitution to integrate  $\int \frac{x}{\sqrt{x^2 - 6x}} dx$ .

c) Convert to an integral involving rational functions (do not integrate):  $\int \frac{\sin x}{2 + \cos x} dx$ .

6. (10 pts) Find the Taylor polynomials.

a)  $T_{4n+1}\left(\frac{x}{1+x^4}\right)$

b)  $T_{2n}(\cos^2(x))$  [Hint: use a trig identity first.]