Math 166: Honors Calculus II Name:________________________

Exam II *Apr. 8, 1999*

There are 6 questions, each with several parts, worth a total of 110 points. Be sure to show all your work and justify all steps.

1. (20 pts) Define the following

   a) $\lim_{x\to a} f(x) = \infty$

   b) $\lim_{n\to\infty} a_n = L$

   c) $f(x) = o(g(x))$ as $x \to a$.

   d) $\sum_{n=1}^{\infty} a_n$ converges.

2. (20 pts) State the following theorems precisely.

   a) Taylor's Theorem with Integral Remainder.

   b) State L'Hôpital's Theorem.

   c) The Limit Comparison Test.

   d) The Integral Test.

3. (20 pts)

a) Express $x^3 \log(1+x)$ in o-notation; include at least three non-zero terms.

b) Simplify $\dfrac{1+x+x^2+o(x^3)}{1-x^2+x^3+o(x^3)}$ into the form $a_0 + a_1 x + a_2 x^2 + o(x^2)$.

c) Let $f(x) = \sin(x)$ and consider $f(x) = T_n f(x; 0) + E_n f(x)$. Find n such that $|E_n f(x)| < 10^{-3}$ for all $x \in [-1, 1]$.

d) Prove that if $|r| < 1$ then $\displaystyle\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

4. (20 pts) Compute the following limits. Be sure to justify each step.

a) $\lim_{x\to 0}(1+h(x))^{1/h(x)}$ where $h(x)$ is a non-constant differentiable function in a neighborhood of 0 and $h(0)=0$.

b) $\lim_{x\to 0}\dfrac{e^{x^2}-\cos(x)}{x^2}$

c) $\lim_{x\to\infty} \sqrt{2x+x^2} - \sqrt{x+x^2}$

d) $\lim_{n\to\infty} \frac{(n+(-1)^n)(n+1)}{3n^2} \quad (n \in)$

5. (15 pts) Compute the sums of the following series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{a^n + (-1)^n}{b^{2n}}$ where $0 < a < b$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n+1} x^n$ for $|x| < 1$

(You may use integration or differentiation term by term.)

6. (15 pts) Test the following series for convergence. Justify your answers.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2}$

c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\log(n)}$