Math 166: Honors Calculus II   Name:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯
Final Exam   *May 6, 1999*

There are 12 pages of problems worth a total of 165 points.

1. (25 pts) Give complete definitions of the following functions, including domains and ranges.

   a) $\log(x)$

   b) $\exp(x)$

   c) a^x for $a > 0$

   d) $\sinh(x)$

   e) $\arcsin(x)$

2. (15 pts) State and prove the First Fundamental Theorem of Calculus.

3. (10 pts)

a) Prove $\dfrac{d}{dx}\arctan(x) = \dfrac{1}{1+x^2}$.

b) Let $f(x) = \dfrac{x+1}{x-1}$. Determine the interval(s) on which $f(x)$ has an inverse and derive a formula for $f^{-1}(x)$.

4. (10 pts)

a) Compute $\dfrac{d}{dx} \displaystyle\int_{\sin(x)}^{\cos(x)} \exp(1 - t^2)\, dt.$

b) Find the maximum value of the function $f(x) = x^{-x}$, $x > 0$.

5. (25 pts) Compute the integrals

a) $\int \dfrac{x}{\sqrt{x+4}}\,dx$

b) $\int \dfrac{1}{x^2+2x}\,dx.$

c) $\int \dfrac{1}{\sqrt{3-2x-x^2}}\,dx.$

d) $\int \dfrac{1}{2 + \cos(\theta)} \, d\theta$

e) $\displaystyle\int_0^\infty e^{-x} \sin(x) \, dx$

6. (15 pts)

   a) State Taylor's Formula with Lagrange Remainder.

   b) Find $T_{2n+1}\left(\dfrac{x}{x^2 + 3}\right)$.

   c) Find $T_7[\log(1 + x^2) - \log(1 + x^3)]$.

7. (20 pts) Compute the following limits.

a) $\lim_{x \to 0} \dfrac{\cos(x^2) - 1}{x^2 \sin^2(x)}$

b) $\lim_{x \to 0^+} \dfrac{1}{x}\left(\dfrac{1}{\sqrt{1-x}} - \dfrac{1}{\sqrt{1+x}}\right)$

c) $\lim_{x \to \infty} \dfrac{\sinh(1+x)}{\cosh(x)}$

d) $\lim_{n \to \infty} \left(n^{100} + (-1)^n\right)^{1/n}$

8. (20 pts) Compute the value of the following infinite series.

a) $\sum\limits_{n=1}^{\infty} \left(\dfrac{1}{n} - \dfrac{1}{n+2} \right)$

b) $\sum\limits_{n=1}^{\infty} \dfrac{2 + (-1)^n}{3^n}$

c) $2x^3 + 4x^5 + 6x^7 + 8x^9 + \cdots$ for $|x| < 1$

d) $\dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{6} - \dfrac{1}{8} + \cdots$ [Hint: Abel's Theorem]

9. (15 pts) Test whether the following series converge absolutely, conditionally, or diverge. Justify your answers.

a) $\sum \dfrac{(-1)^n}{n\log(n)^s}$ for $s > 1$.

b) $\sum_{n=1}^{\infty} (-1)^n \left(1 + \dfrac{1}{n}\right)^n$

c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n}{n^2 + 1}$

10. (10 pts) Determine the interval of convergence for the following power series.

a) $\sum_{n=0}^{\infty} (1 + (-3)^n)x^n$

b) $\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{n3^n}$