Math 166: Honors Calculus II   Name:_____

Exam I   *Mar. 2, 2000*

There are 8 questions, most with several parts, worth a total of 110 points.
Be sure to show all your work and justify all steps.

1. (25 pts) Define each of the following completely and precisely.

   a) The natural logarithm function, $\log(x)$.

   b) The natural exponential function, $\exp(x)$.

   c) a^x for $a > 0$.

   d) $\sinh(x)$ and $\cosh(x)$

   e) $T_n f(x; a)$

2. (10 pts)

   a) State the Fundamental Theorem of Calculus.

   b) Prove this theorem for a continuous function on $[a, b]$ using the Mean Value Theorem.

3. (10 pts)

   a) State Taylor's Formula with the remainder (or error) term expressed as an integral.

   b) Prove the error term $E_n f(x)$ has the form stated above by considering the function $F(t) = T_n f(x;t)$. (Show that $E_n f(x) = F(x) - F(a) = \ldots$)

4. (15 pts) Compute the following derivatives.

a) $\dfrac{d}{dx}\displaystyle\int_{1/x^2}^{x^2} \log(\cos(\sqrt{t}))\,dt$

b) $\dfrac{d}{dx}(1+x^3)^{(1+x^5)}$

c) $(f^{-1})'(1)$ if $f(x) = x + e^x$.

5. (15 pts) Compute the following integrals.

a) $\int \dfrac{\sqrt{x-1}}{x}\, dx$

b) $\int (x\cos(x))^2\, dx$

c) $\int \dfrac{x}{\sqrt{x^2-x}}\, dx$

6. (10 pts)

a) Give the partial fraction decomposition of $\dfrac{1}{x^3 - 1}$.

(Solve for the constants.)

b) Use an appropriate substitution to transform

$$\int \frac{(\sin(x) + 1)^2}{\cos(x) + 3}\, dx$$

into the integral of a rational function of u.

(Simplify as much as possible, but do not integrate.)

7. (15 pts) Find the Taylor polynomials.

a) $T_3 \tan(x)$

b) $T_{2n}(\log(1 - x^2))$

c) $T_{3n}\left(\dfrac{1 + x^n}{1 + x^{2n}}\right)$

8. (10 pts)

Use the Taylor polynomial of degree 3 for e^x to find an approximate value for the integral

$$\int_{0.5}^{1.0} \frac{e^x}{x} \, dx$$

and give an estimate for the error.