There are 7 pages of questions worth a total of 110 points. Be sure to show all your work and justify all steps.

(1) (25 pts) Define the following

  a) $\lim_{x \to \infty} f(x) = L$ and $\lim_{x \to a} f(x) = \infty$.

  b) $f(x) = o(g(x))$ as $x \to a$.

  c) Convergence and divergence of a series.

  d) Absolute and conditional convergence.

(2) (25 pts) State the following theorems precisely.

a) L'Hôpital's Rule

b) The Limit Comparison Test

c) The Integral Test

d) The Root Test

e) Leibniz's Rule

(3) (15 pts) Compute the following limits, justifying each step.

a) $\lim\limits_{x \to 0} \dfrac{x^2 \sin(x) - x \sin(x^2)}{\sin(x^5)}$

b) $\lim\limits_{n \to \infty} (n+1)^c - n^c$ where $c < 1$

c) $\lim\limits_{x \to 1} x^{1/(1-x)}$

(4) (10 pts) Prove that if $|x| < 1$ then $\displaystyle\sum_{n=k}^{\infty} x^n = \frac{x^k}{1 - x}$.

(5) (10 pts) Compute the sums of the following series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2^n - 1)(2^n + 1)}{5^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

(6) (25 pts) Test the following series for convergence or divergence. If appropriate, determine whether the convergence is absolute or conditional.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1+1/n}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

c) $\displaystyle\sum_{n=1}^{\infty} n(-\pi)^{-n}$

d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log(n))^2}$

e) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n} + (-1)^n n}{n\sqrt{n}}$