Math 166: Honors Calculus II Name:__________________________

Final Exam *May 12, 2000*

There are 9 problems on 11 pages worth a total of 165 points.

(1) (30 pts) Give complete definitions for the following.

a) $\log(x)$ and $\exp(x)$

b) b^a for $a, b \in \mathbb{R}$, $b > 0$

c) $\sinh(x)$ and $\cosh(x)$

d) $T_n f(x; a)$

e) $\sum_{n=1}^{\infty} a_n$

f) $\int_{-\infty}^{\infty} f(x)\, dx$

(2) (30 pts) State the following theorems precisely.

a) The First Fundamental Theorem of Calculus.

b) Taylor's Theorem with Remainder

c) L'Hôpital's Rule

d) The Ratio Test

e) Uniform Convergence of Power Series

f) Termwise Integration of Power Series

(3) (15 pts)

a) Compute $\dfrac{d}{dx}\displaystyle\int_{\log(x)}^{\tan(x)} \exp(t\sqrt{1+t^2})\,dt.$

b) Find the maximum value of the function $f(x) = x^{1/x}$, $x > 0$.

c) Show that $\dfrac{d}{dx}\arcsin(x) = \dfrac{1}{\sqrt{1-x^2}}$

(4) (20 pts) Compute the integrals

a) $\displaystyle\int \frac{dx}{1+\sqrt{x}}$

b) $\displaystyle\int_0^\infty xe^{-x}\,dx$

c) $\displaystyle \int \frac{x+1}{x^4+x^2}\,dx$

d) $\displaystyle \int \frac{dx}{1+\sin(x)+\cos(x)}$

(5) (20 pts) Compute the limits.

a) $\lim_{x\to 0} \frac{\cos(2x^3) - \exp(-2x^6)}{x^6 \log(1+x^6)}$

b) $\lim_{x\to\infty} \left(x^2 - x^3 \sin \frac{1}{x}\right)$

c) $\lim\limits_{x \to 1} \dfrac{3x^4 - 8x^3 + 12x - 7}{x^4 - 4x + 3}$

d) $\lim\limits_{n \to \infty} \dfrac{2^n + (-1)^n}{2^{n+1} + (-1)^{n+1}}$

(6) (15 pts) Sum the series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+3n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{a^n+(-1)^{n+1}}{(a+1)^{2n}} \quad a>0$

c) $\displaystyle\sum_{n=1}^{\infty} n^2 x^n \quad |x|<1$

(7) (15 pts) Test the series for convergence.

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1-n}{1+n}\right)^{n}$

b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^{n}}{\sqrt{n^{2}-1}}$

c) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n}}{\sqrt{n}2^{\sqrt{n}}}$

(8) (10 pts) Determine the interval of convergence for the power series.

a) $1 + \frac{1}{2}x + \frac{1}{4 \cdot 3}x^2 + \frac{1}{6 \cdot 5 \cdot 4}x^3 + \frac{1}{8 \cdot 7 \cdot 6 \cdot 5}x^4 + \cdots$

[*Hint:* Rewrite the coefficients using factorials.]

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2} x^{2n}$

(9) (10 pts)

a) Find a power series for $\int_0^x e^{-t^2}\,dt$.

b) Use the series in part a) to calculate an approximation to $\int_0^1 e^{-t^2}\,dt$ with error $< 10^{-4}$. [*Hint:* The series is alternating.]