Math 166: Honors Calculus II Name:_____
Quiz 8 *March 30, 2000*

1. Determine whether each of the following sequences converges or diverges and find the limit of each convergent sequence.

   a) $a_n = (-1)^n\Big(1 - \dfrac{1}{n}\Big)$

   b) $a_n = \dfrac{\sin(n\pi/2)}{n\pi/2}$

   c) $a_n = \Big(1 + \dfrac{c}{n}\Big)^n$

2. Let $0 < b < a$.

a) Prove that $\lim_{n\to\infty} n^b \sum_{k=1}^{n} \frac{1}{(n+k)^{a+1}} = 0.$ [Hint: $\frac{1}{n+k} < \frac{1}{n}$]

b) Prove that $\lim_{n\to\infty} n^a \sum_{k=1}^{n} \frac{1}{(n+k)^{a+1}} = \frac{1}{a}\big(1 - \frac{1}{2^a}\big).$ [Hint: $\int_0^1 f(x)\,dx$]