DEPARTMENT OF MATHEMATICS
COMPETITION FOR THE AUMAN PRIZE
2000

Any student enrolled in MATH 166, Honors Calculus II, may enter. Each competitor must solve the problems independently under the honor code. At least two problems must be solved to qualify. The student who provides the most complete solutions will win a cash prize of $200. Solutions must be submitted to Prof. Dennis Snow, CCMB 313, by noon Friday, March 24.

(1) Let $f(x)$ be differentiable at $x = a$, and $f(a) \neq 0$. Evaluate

$$\lim_{n \to \infty} \left[\frac{f(a + 1/n)}{f(a)} \right]^n$$

(2) Prove that there is no continuous function f with domain the closed interval $[0, 1]$ and range the open interval $(0, 1)$.

(3) Calculate $\lim_{x \to \infty} x \int_0^x e^{t^2 - x^2} dt$.

(4) Determine how far a stack of n identical books can be made to project over the edge of a table by stacking them one on top of the other. Determine the theoretical limit of this overhang as $n \to \infty$.

(5) Show if there are $n \geq 2$ people in a room, then two of them know the same number of people among those present. (Assume that if A knows B, then B knows A).