GABRIEL'S HORN

MATH 166: HONORS CALCULUS II

"Gabriel's horn" is the surface generated by revolving the graph of $y = 1/x$ from 1 to ∞ about the x-axis.

To calculate the volume of the solid of revolution bounded by the horn, note that the solid is filled out by infinitesimally thin cylinders of radius $1/x$ and width dx. So, over any finite interval $[1, a]$ the volume is given by

$$\int_1^a \pi \left(\frac{1}{x}\right)^2 dx = \pi\left(1 - \frac{1}{a}\right)$$

As $a \to \infty$, the volume approaches π which is a resonable definition of the volume even though the solid has infinite extent.

On the other hand, it can be shown that the surface of Gabriel's horn over any finite interval $[1, a]$ is given by

$$\int_1^a 2\pi \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx > \int_1^a 2\pi \frac{1}{x}\, dx = 2\pi \log(a)$$

Since $2\pi \log(a) \to \infty$ as $a \to \infty$, the surface area of Gabriel's horn cannot be assigned a finite value. This counter-intuitive situation could be summarized by saying that Gabriel's horn holds a finite amount of paint, but not enough to cover its inside surface!

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