Math 196. Final Exam, May 7, 2001 Name ______________________________

1) (12 pts.) You are given the function $f(x) = 2x^3 - 24x^2$.

i) Find the critical numbers for f.

ii) Determine the intervals over which f is increasing and those over which f is decreasing.

2) (16 pts.) On October 12, 1999, the New York Times (in the Science Times Section) reported that a team of astronomers using a telescope on Mauna Kea, Hawaii, had discovered a moon orbiting the asteroid Eugenia (in the main asteroid belt between Mars and Jupiter). The moon is about 13 kilometers across, and orbits Eugenia in a nearly circular orbit at a distance of 1120 kilometers once every 4.7 days. (Use MKS in your computations below.)

i) Compute (or, more appropriately, estimate) the speed with which the moon orbits Eugenia.

ii) Compute (or, more appropriately, estimate) the mass of Eugenia (in kilograms).

3) (14 pts.) Give an explanation of the formula $F_P = \frac{8\kappa^2 m}{L}\frac{1}{r_P^2}$ in the context of the orbit of Eugenia's moon. Include a complete discussion of the underlying physical situation, draw an appropriate diagram, and define the constants κ, L, and m. Use the data from the previous problem to compute κ for the orbit and express the gravitational force (in newtons) with which Eugenia pulls on its moon in terms of the mass m of this moon.

4) (16 pts.) The Tacoma Narrows Bridge was completely rebuilt in the years 1948 to 1950. The data for the new bridge is as follows. It has a total length of 5,000 feet and a center span of 2,800 feet. Its 2 main cables support a single deck that carries 4 lanes of automobile traffic. The dead load is 8,680 pounds per foot. Assume that it is designed for a live load capacity of 4,000 pounds per foot and that the sag in the cable over the center span is 280 feet.

i) Consider one of the main cables over the center span at a point where it meets one of the towers. Compute the tension T of the cable at that point and compute the angle α that the cable makes with the horizontal at that point.

ii) Compute the contribution of this one cable to the compression forces on the tower

5) (24 pts.) The metal rim of a wagon wheel of radius 0.5 meters and mass of 2 kilograms and a bowling ball of 10.8 centimeters in diameter and a mass of 5 kilograms are poised for a race down a ramp that is 10 meters long and makes an angle of $30°$ with the horizontal. The index of inertia of the rim is given by the expression mr^2 and that of the ball by $\frac{2}{5}mr^2$. Under the assumption that the friction produced by the ramp is precisely that force that rotates the two objects do the following:

i) Draw pictures of the situations and use them as a guide towards your solutions of the problems below.

ii) In each case compute the frictional force on the object f in terms of the angular acceleration.

a) For the rim:

b) For the bowling ball:

iii) In each case compute the total force F on the object and use $F = ma$ to calculate the linear acceleration a of the motion of each of the two objects down the ramp.

a) For the rim:

b) For the bowling ball:

iv) Assuming that both the rim and the bowling ball start from rest (without being given a push), which will win the race?

6) (28 pts.) A travel agent offers the following group deal for a bus tour to the Kentucky Derby. It will only accept a booking of at least 25 people. For precisely this number the cost is 120 dollars per person. For each additional person up to a maximum of 40 people (this is the capacity of the buses that the agency operates), there is a reduction of 2 dollars per person. So the number of people going on the trip is some number x between 25 and 40. It costs the agency 2000 dollars (fixed cost) plus 18 dollars per person to operate a bus down to Louisville.

i) Express the price per person as a function of the number of travellers x.

ii) For what number of travellers is the revenue a maximum?

iii) Express the cost to the agency as a function of the number of travellers.

iv) For what number of travellers is the profit at maximum level?

v) What changes in the pricing strategy will result in a situation where the number of travellers that maximizes the profit coincides with the capacity of the bus? What cost cutting strategy has the same effect?

vi) Suppose that the trip takes place at the maximum capacity of 40 persons. Notice that in this case the cost of the trip is $(120 - 15 \cdot 2) = 90$ dollars. What is the total consumer surplus under the assumption that all the travellers would have been willing to pay 120 for the trip.

FORMULAS

$T_0 = T_x \cos\theta \quad wx = T_x \sin\theta \quad T_d = \sqrt{(\frac{d}{2s})^2 \ + 1} \quad \tan\alpha = \frac{2s}{d}$

$\theta(t) = \frac{1}{r}s(t) \quad \omega(t) = \frac{1}{r}v(t) \quad \alpha(t) = \frac{1}{r}a(t)$

Torque = (Index of Inertia)(Angular Acceleration)

$F_P = \frac{4\pi^2 a^3 m}{T^2}\ \frac{1}{r_P^2},\ F_P = \frac{8\kappa^2 m}{L}\ \frac{1}{r_P^2},\ \frac{a^3}{T^2} = \frac{GM}{4\pi^2},\ M = \frac{4\pi^2 a^3}{GT^2},$

$A_p = A_0(1+\frac{r}{n})^p \quad A(t) = A_{nt} = A_0(1+\frac{r}{n})^{nt}$

$A(t) = A_0 e^{rt} \quad S_p = \frac{12}{r}A_0(1+\frac{r}{12})((1+\frac{r}{12})^p - 1)$

$PV_p = \frac{12B}{r}[1-(1+\frac{r}{12})^{-p}] \quad PV_p = \frac{2C}{r}[1-(1+\frac{r}{2})^{-p}]$

$p(t) = p(0)e^{kt} \quad e_D(p) = p\frac{D'(p)}{D(p)} \quad e_S(p) = p\frac{S'(p)}{S(p)}$

$\Pi(x) = R(x) - C(x)$

$G = 6.67 \times 10^{-11} \frac{\text{meters}^3}{\text{kilograms}\cdot\text{seconds}^2}$