Name: ______________________________ **Instructor:** Heide Gluesing-Luerssen

Mathematics 214: Introduction to Statistics
Spring Semester 1998
Exam 1
February 6, 1998

This Examination contains 15 multiple choice questions 6 points worth each. You start with 10 points, and the highest possible score is 100. Fill in your answers on this cover sheet by placing an X through one letter for each problem. Calculators, books, and notes are not allowed.

1	a	b	c	c m d	e	
2	a	b	c	c m d	e	
3	a	b	c	c m d	e	
4	a	b	c	c m d	e	
5	a	b	c	c m d	e	
6	a	b	c	c m d	e	
7	a	b	c	c m d	e	
8	a	b	c	c m d	e	

hline 9	a	b	c	d	e	
10	a	b	c	d	e	
11	a	b	c	d	e	
12	a	b	c	d	e	
13	a	b	c	d	e	
14	a	b	c	d	e	
15	a	b	c	d	e	

Total $\boxed{}$

Sign the pledge:

"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: ________________________________

GOOD LUCK

1) You roll a fair die twice in a row. What is the probability that the sum of the upper face numbers is 8?

a) $\frac{1}{6}$ **b)** $\frac{\binom{12}{8}}{36}$ **c)** $\frac{1}{12}$ **d)** $\frac{5}{36}$ **e)** $\frac{1}{\binom{36}{8}}$

2) The set of six letters $\{A, B, C, D, E, F\}$ is used to construct 4-letter words. Repetitions of letters are allowed and the order counts, i. e., the word $ABDD$ is different from the word $BDAD$. Assume that the letters are chosen randomly, i. e. that any letter chosen from the set $\{A, B, C, D, E, F\}$ is equally likely. What is the probability that a random word has all its 4 letters different?

a) $\frac{\binom{6}{4}}{6^4}$ **b)** $\frac{P_4^6}{6^4}$ **c)** $\frac{1}{6^4}$ **d)** $\frac{4!}{6!}$ **e)** $\frac{\binom{6}{4}}{6!}$

3) There is a well-shuffled standard card deck of 52 cards. What is the probability that a poker hand of five cards dealt randomly from the deck contains exactly 3 aces.

a) $\frac{\binom{4}{3}\cdot\binom{48}{2}}{\binom{52}{5}}$ **b)** $\frac{5\cdot 4\cdot 3\cdot 47^2}{52^5}$ **c)** $\frac{\binom{4}{3}\cdot 48^3}{\binom{52}{5}}$ **d)** $\frac{\binom{4}{3}+\binom{48}{2}}{\binom{52}{5}}$ **e)** $\frac{4^3\cdot 47^2}{52^5}$

4) Assume A and B are events, i. e. subsets of a sample space S. Let P be a probability defined on S. Assume that $P(A) = 0.45$, $P(B) = 0.15$, and $P(A \cap B) = 0.03$. Calculate $P(A \,|\, B)$.

a) $\frac{1}{2}$ **b)** 0.45 **c)** $\frac{1}{3}$ **d)** 0.03 **e)** $\frac{1}{5}$

5) In a certain company 60% of the employees are female and the remaining 40% are, of course, male. 30% of the female employees and 40% of the male employees are younger than 35 years of age. What fraction of all the employees of that company are younger than 35 years of age?

a) 0.32 **b)** 0.5 **c)** 0.36 **d)** 0.7 **e)** 0.34

6) There is a box of nails. Assume that $\frac{2}{3}$ of these nails were produced by a machine (call it A) that is known to produce defective nails with probability 0.01, and the remaining $\frac{1}{3}$ of the nails were produced by a machine (call it B) that is known to produce defective nails with probability 0.03. If a nail is chosen randomly from the box and it is found to be defective, what is the probability it was produced by machine A?

a) 1 **b)** 0.6 **c)** 0.4 **d)** 0.5 **e)** 0.2

7) At a certain institute 55% of the members have cookies in the afternoon, 40% of the members have tea in the afternoon, and 20 % of the members have neither. What fraction of the members like both cookies and tea?

a) 40% **b)** 15% **c)** 20% **d)** 95% **e)** 80%

8) A company produces light bulbs and ships boxes consisting of 40 bulbs. Before shipping, a random sample of 6 bulbs is taken from the box and inspected — if any are defective, the box is not shipped. Assuming the box contains 8 defective bulbs, what is the probability that the box won't be shipped, i. e., that the random sample of 6 bulbs contains at least one defective.

a) $\dfrac{32^6 - 8^6}{40^6}$ **b)** $\dfrac{\binom{8}{6}}{\binom{40}{6}}$ **c)** $\dfrac{\binom{32}{6}}{\binom{40}{6}}$ **d)** $1 - \dfrac{\binom{32}{6}}{\binom{40}{6}}$ **e)** $\dfrac{\binom{8}{1} \cdot \binom{32}{8}}{\binom{40}{8}}$

9) 39 % of a given population has type A blood. A specific blood typing test predicts type A blood for 37% of the population. If a person from the population has type A blood, then the test will predict with probability 0.91 that the person's blood is type A. What is the probability that a certain person actually has type A blood given that the test predicts type A blood for that person?

a) $0.91 \cdot 0.38$ **b)** $\dfrac{0.91 \cdot 0.39}{0.37}$ **c)** $\dfrac{0.91 \cdot 0.37}{0.39}$ **d)** 1 **e)** $0.91 \cdot 0.37$

10) Assume that A and B are events of a sample space S. The probabilities $P(B) = 0.4$, $P(A \mid B) = 0.3$, and $P(A \mid \bar{B}) = 0.6$ are known. What is $P(A)$?

a) 0.48 **b)** 0.12 **c)** 0.24 **d)** 0.7 **e)** 0.9

11) Assume A and B are disjoint events, i. e., A and B are subsets of a sample space S and the intersection $AB = A \cap B = \emptyset$. Let P be a probability defined for the sample space S. If further A and B are independent events, then it is always true that $P(A) \cdot P(B) =$

a) $P(A) + P(B)$ **b)** $\frac{1}{2}$ **c)** 1 **d)** 0 **e)** $1 - P(A) - P(B)$

12) Let X be a discrete random variable which takes the values -1, 2, and 4. Its probability function p is given as $p(-1) = 0.4$, $p(2) = 0.3$, and $p(4) = 0.3$. What is $E(X)$, i. e., what is the expected value of X?

a) 2 **b)** 1 **c)** 5 **d)** $\sqrt{1.4}$ **e)** 1.4

13) How many ways can 3 different books be given to 7 persons, if each person is allowed to receive several books?

a) 7^3 **b)** 3^7 **c)** $\binom{7}{3}$ **d)** P_3^7 **e)** $3 \cdot 7$

14) Let X be a discrete random variable which takes the values -1, 2, 3, and 4. The probability function of X is given as $p(-1) = 0.1$, $p(2) = 0.2$, $p(3) = 0.1$, and $p(4) = 0.6$. What is the standard deviation of X?

a) 2.4 **b)** 2 **c)** $\sqrt{2.4}$ **d)** 1.2 **e)** $\sqrt{3}$

15) Of the people entering a blood bank to donate blood, 30% have type O^+ blood. For the next three people entering, let X denote the number with O^+ blood. What is $P(X = 2)$?

a) $3 \cdot (0.3)^2 \cdot 0.7$ **b)** $3 \cdot (0.7)^2 \cdot 0.3$ **c)** $(0.3)^2$ **d)** $(0.7) \cdot 0.3$ **e)** $(0.3)^2 \cdot 0.7$