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Mathematics 214: Introduction to Statistics
Spring Semester 1998
Exam 2
March 4, 1998

This Examination contains 16 questions 6 points worth each. You start with 4 points, and the highest possible score is 100. Fill in your answers on this cover sheet by placing an X through one letter for each problem except problems 7 and 8. Write the answers for 7 and 8 in the corresponding empty boxes of the list below. Calculators, books, and notes are not allowed.

1	a	b	c	c md	e	
2	a	b	c	c md	e	
3	a	b	c	c md	e	
4	a	b	c	c md	e	
5	a	b	c	c md	e	
6	a	b	c	c md	e	
7						
8						

hline 9	a	b	c	d	e	
10	a	b	c	d	e	
11	a	b	c	d	e	
12	a	b	c	d	e	
13	a	b	c	d	e	
14	a	b	c	d	e	
15	a	b	c	d	e	
16	a	b	c	d	e	

Total

Sign the pledge:

"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: ________________________________

GOOD LUCK

1. Let X be a discrete random variable with $V(X) = 4$, i. e. variance equal to 4. What is the variance of $6X + 10$?

a) 144 **b)** 24 **c)** 34 **d)** 154 **e)** 400

2. 10% of the nails produced by a certain machine have defects. If 20 nails are randomly selected one at a time for inspection, as the machine produces them, find the variance of the number of inspected nails that have defects.

a) 2 **b)** 1.8 **c)** $\sqrt{1.8}$ **d)** $(1.8)^2$ **e)** 18

3. Let Y denote a random variable having a geometric distribution, with probability of success on any trial given by $p = 0.3$. Find $P(Y = 4)$.

a) $(0.7)^3(0.3)$ **b)** $1 - (0.3)^3(0.7)$ **c)** $1 - (0.7)^3(0.3)$ **d)** $1 - \binom{3}{1}(0.3)(0.7)^3$

e) $1 - 0.3 - (0.7)(0.3) - (0.7)^2(0.3)$

4. Sixty percent of a population of consumers is reputed to prefer Brand A toothpaste. If a group of consumers is interviewed, what is the probability that exactly 5 people must be interviewed before a consumer is encountered who prefers Brand A?

a) $4(0.4)^4(0.6)$ **b)** $\binom{4}{1}(0.4)^4(0.6)$ **c)** $5(0.4)^4(0.6)$ **d)** $(0.6)^4(0.4)$

e) $(0.4)^4(0.6)$

5. 5% of the lightbulbs manufactured on a certain assembly line are defective. If bulbs are randomly selected one at a time and tested, find the probability that the third nondefective lightbulb is found on the fifth trial.

a) $\binom{5}{3}(0.05)^2(0.95)^3$ **b)** $\binom{4}{2}(0.05)^2(0.95)^3$ **c)** $(0.05)^3(0.95)^2$

d) $\binom{4}{2}(0.05)^3(0.95)^2$ **e)** $(0.05)^2(0.95)^3$

6. A certain manufacturer advertises batteries that will run for an average of 150 minutes, with a standard deviation of 10 minutes. Find the smallest interval, which contains at least 75% of the performance periods for batteries of this type? periods for batteries

a) $50 \le Y \le 250$ **b)** $140 \le Y \le 160$ **c)** $110 \le Y \le 190$ **d)** $130 \le Y \le 170$

e) $145 \le Y \le 155$

7. Consider a binomial experiment for $n = 20$ and $p = 0.05$. Use the appended table to find $P(Y \ge 3)$. **Write your answer on the cover sheet!**

8. Let Y be a discrete random variable having a Poisson distribution with mean $\lambda = 1$. Use the appended table to find $P(3 \le Y \le 5)$. **Write your answer on the cover sheet!**

9. The number of telephone calls coming into a central switchboard of an office building has a Poisson distribution and averages 7 per minute. Find the probability that at least one call will arrive within a given two minute period.

a) $1 - e^{-1}$ **b)** $1 - e^{-14}$ **c)** $e^{-14} + 14e^{-14}$ **d)** $2(1 - e^{-7})$

e) $(1 - e^{-7})^2$

10. The proportion of time X that a certain computer system is in operation during a week is a random variable with probability density function

$$f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find $E(X)$.

a) 4 **b)** 1 **c)** $\frac{4}{5}$ **d)** $\frac{2}{3}$ **e)** 0

11. For the computer in the last problem, find the probability that the computer is in operation at most half of the week, i. e., $P(X \leq 0.5)$.

a) $\frac{15}{16}$ **b)** $\frac{1}{40}$ **c)** $\frac{1}{4}$ **d)** $\frac{3}{64}$ **e)** $\frac{1}{16}$

12. Let X be a random variable with the uniform distribution

$$f(x) = \begin{cases} \frac{1}{3} & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

as its probability density function. Find $V(X)$.

a) $\frac{3}{4}$ **b)** $\frac{1}{4}$ **c)** $\frac{3}{2}$ **d)** $\frac{9}{2}$ **e)** 9

13. Telephone calls coming into a certain switchboard follow a Poisson distribution. It is known that during a given 5-minute period, one call arrived at the switchboard. Find the probability that the call arrived within the first minute of this period.

a) $\frac{4}{5}$ **b)** $5e^{-5}$ **c)** $\frac{1}{5}e^{-5}$ **d)** $\frac{1}{4}$ **e)** $\frac{1}{5}$

14. Assume that the number of fatal accidents on scheduled domestic passenger airlines follows a Poisson distribution with a mean of one fatal accident every 41 days. Then the probability distribution for the interaccident times (time between fatal accidents) on scheduled domestic passenger flights is a(n):

a) Poisson distribution **b)** binomial distribution **c)** exponential distribution

d) normal distribution **e)** uniform distribution

15. An engineer has observed that the gap times between vehicles passing a certain point on a highway have an exponential distribution with a mean of 10 seconds. Find the probability that the next gap observed will be no longer than 1 minute.

a) e^{-6} **b)** $1 - e^{-6}$ **c)** $1 - \frac{1}{10}e^{-6}$ **d)** $1 - e^{-60}$ **e)** $1 - e^{-1}$

16. Let the random variable Y possess a probability density function

$$f(y) = \begin{cases} cy & 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find c.

a) $\frac{1}{4}$ **b)** 2 **c)** $\frac{1}{2}$ **d)** 4 **e)** 1