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Mathematics 214: Introduction to Statistics
Spring Semester 1998
Final Exam
May 4, 1998

This Examination contains 30 questions 5 points worth each. The highest possible score is 150 points. Fill in your answers on this cover sheet by placing an X through one letter for each problem. Books and notes are not allowed.

1	a	b	c	c m d	e	
2	a	b	c	c m d	e	
3	a	b	c	c m d	e	
4	a	b	c	c m d	e	
5	a	b	c	c m d	e	
6	a	b	c	c m d	e	
7	a	b	c	c m d	e	
8	a	b	c	c m d	e	
9	a	b	c	c m d	e	
10	a	b	c	c m d	e	
11	a	b	c	c m d	e	
12	a	b	c	c m d	e	
13	a	b	c	c m d	e	
14	a	b	c	c m d	e	
15	a	b	c	c m d	e	

hline 16	a	b	c	d	e	
17	a	b	c	d	e	
18	a	b	c	d	e	
19	a	b	c	d	e	
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25	a	b	c	d	e	
26	a	b	c	d	e	
27	a	b	c	d	e	
28	a	b	c	d	e	
29	a	b	c	d	e	
30	a	b	c	d	e	

Total | |

Sign the pledge:

"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: ______________________________________

GOOD LUCK

1. A fair die is tossed 5 times. What is the probability that a "five" comes up exactly 3 times?

a) $\dfrac{\binom{6}{1}\binom{6}{5}}{6!}$ **b)** $\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$ **c)** $\binom{5}{3}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$ **d)** $\binom{5}{3}\left(\frac{1}{6}\right)^3$ **e)** $\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$

2. Five employees of a firm are ranked from 1 to 5 based on their ability to program a computer. Three of these employees are selected to fill three identical programming jobs. If all possible choices of three (out of five) are equaly likely, what is the probability that the highest-ranked employee among those selected has rank 4?

a) $\dfrac{3}{10}$ **b)** $\dfrac{1}{5}$ **c)** $\dfrac{P_2^5}{5!}$ **d)** $\dfrac{P_3^5}{5^3}$ **e)** $\dfrac{2}{5^3}$

3. How many 7-digit phone-numbers be constructed using the digits 0, 1, . . . , 9, if the first digit has to be nonzero.

a) $9 \cdot 10^7$ **b)** 10^7 **c)** 9^7 **d)** $10^7 - 9^7$ **e)** $9 \cdot 10^6$

4. A discrete random variable takes the values -1, 3, and 5 with the probabilities $P(X = -1) = 0.5$, $P(X = 3) = 0.3$, and $P(X = 5) = 0.2$. Calculate $E(X^2)$.

a) 8.2 **b)** 0.72 **c)** 7.2 **d)** 0.38 **e)** 1.4

5. If 30% of the persons donating blood at a clinic have O^+ blood, find the probability that one has to wait for exactly the 10th donor to get the 3rd O^+-donation.

a) $(0.7)^7$ **b)** $\binom{9}{2}(0.3)^3(0.7)^7$ **c)** $\binom{10}{3}(0.3)^3(0.7)^7$ **d)** $\binom{9}{2}(0.7)^3(0.3)^7$ **e)** $3(0.3)^3(0.7)^7$

6. You roll a fair die three times in a row. What is the probability that the sum of the upper face numbers is 6?

a) $\frac{3}{216}$ **b)** $\frac{13}{216}$ **c)** $\frac{10}{216}$ **d)** $\frac{7}{216}$ **e)** $\frac{12}{216}$

7. A blood test for X, a certain disease, has the following accuracy:

	test result	
	positive	negative
patient with X	0.99	0.01
patient without X	0.02	0.98

That is, if a person has X, then the probability of a positive test result is 0.99, etc. The disease rate for X in the general population is 0.001. If a person tests positive for X, what is the probability that the person has X?

a) 0.009 **b)** 0.09 **c)** 0.25 **d)** 0.047 **e)** 0.99

8. Let S be a sample space and A, B events associated with S. Assume that $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$. Calculate $P(\overline{B} \,|\, A)$.

a) 0.28 **b)** $\frac{0.28}{0.4}$ **c)** $\frac{1}{4}$ **d)** $\frac{1}{2}$ **e)** $\frac{3}{4}$

9. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{4}(x^2 - 1), & 1 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $E(X)$.

a) $\frac{27}{16}$ **b)** $\frac{15}{16}$ **c)** $\frac{7}{4}$ **d)** $\frac{9}{16}$ **e)** 1

10. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate $V(X)$.

a) $\frac{39}{4}$ **b)** 0 **c)** $\frac{12}{5}$ **d)** $\frac{3}{20}$ **e)** $\frac{9}{10}$

11. Let X be as in problem 10. Calculate $P(X \geq 1)$.

a) 1 **b)** $\frac{7}{8}$ **c)** $\frac{1}{8}$ **d)** $\frac{29}{32}$ **e)** $\frac{1}{2}$

12. The number of telephone calls coming into a central switchboard of an office building have a Poisson Distribution and average 5 per minute. Find the probability that at least 2 calls will arrive in a given 1-minute period.

a) $e^{-5} + 5e^{-5}$ **b)** $2e^{-5}$ **c)** $1 - e^{-5} - 5e^{-5} - \frac{25}{2}e^{-5}$ **d)** $1 - e^{-5} - 5e^{-5}$ **e)** $\frac{25}{2}e^{-5}$

13. A telephone call arrived at a switchboard at a random time within a 1-minute interval. The switchboard was fully busy for 15 seconds into this 1-minute period. Find the probability that the call arrived when the switchboard was not fully occupied.

a) e^{-4} **b)** $\frac{3}{4}$ **c)** $\frac{1}{4}$ **d)** $1 - e^{-4}$ **e)** e^{-1}

14. The service times at teller windows in a bank were found to follow an exponential distribution with a mean of 4 minutes. A customer arrives at 2:00 p.m. Find the probability that she will still be there at 2:03 p.m.

a) $1 - e^{-\frac{3}{4}}$ **b)** $\frac{1}{4}e^{-\frac{3}{4}}$ **c)** $1 - \frac{3}{4}e^{-\frac{3}{4}}$ **d)** $\frac{1}{3}e^{-\frac{1}{4}}$ **e)** $e^{-\frac{3}{4}}$

15. Let X be a random variable with a normal probability density function having mean 5 and variance 9. Find $P(X \leq 8)$.

a) 0.3413 **b)** 0.8413 **c)** 0.1587 **d)** 0.6293 **e)** 0.1293

16. A fair coin is tossed 2,000 times. What is the approximate probability that 1,050 or more "heads" come up?

a) 0.9864 **b)** 0.4864 **c)** 0.0136 **d)** 0.0985 **e)** 0.4015

The following table is to be used for the questions 17) – 19).

		X			
		-1	0	2	
	0	0.2	0.1	0.05	
Y	1	0.25	0.1	0	
	2	0.15	0.1	0.05	

17. Find $P(X = 2 \,|\, Y = 0)$.

a) 0.5 **b)** $\frac{1}{3}$ **c)** $\frac{1}{7}$ **d)** 0.05 **e)** 0.45

18. Find $E(XY)$.

a) -0.45 **b)** -0.35 **c)** 1 **d)** 0.25 **e)** -0.2

19. Find $V(X)$.

a) 0.84 **b)** 0.6 **c)** 1.16 **d)** 1 **e)** 1.4

20. Let X, Y be independent random variables with $E(X) = 3$, $E(Y) = 4$ and $V(X) = 9$, $V(Y) = 25$. Then the covariance $Cov(X,Y)$ is

a) 3 **b)** -3 **c)** 0 **d)** 27 **e)** 15

21. Let X, Y be random variables with $V(X) = 9$, $V(Y) = 25$, and $Cov(X,Y) = 20$. Find $V(3X - Y)$.

a) 22 **b)** -18 **c)** -58 **d)** 42 **e)** -14

22. The service times for customers coming through a certain checkout counter are independent random variables with a mean of 2.5 minutes and a variance of 1 minute. Approximate the probability that 50 customers can be serviced in less than 2 hours.

a) 0.5398 **b)** 0.2389 **c)** 0.2611 **d)** 0.7611 **e)** 0.0398

23. For an aptitude test for a certain job history shows scores to be normally distributed with a variance of 225. 20 applicants are to take the test. Find the approximate probability that the sample variance of the test scores will be less than 140.

a) 0.1 **b)** 0.9 **c)** 0.975 **d)** 0.025 **e)** 0.05

24. A random poll of the adults in the U.S.A. has been conducted to see how many people favor football as a sport. The result was that 36% of the 1091 adults sampled list football as their favorite sport. Find a 95% confidence interval for the true fraction of the U.S. adults who favor football.

a) (0.34, 0.38) **b)** (0.35, 0.37) **c)** (0.33, 0.39) **d)** (0.32, 0.4) **e)** (0.31, 0.41)

25. The temperature of the water coming out of a certain power plant is specified by law to be less than or equal to 75 degrees Fahrenheit. The sample mean for a random sample of 16 temperatures taken on a given day is 77 degrees. The sample variance is 9. Assume that the temperature measurements have a normal distribution. What is the smallest significance level with which we can reject the null hypothesis that the temperature is less than 75?

a) 0.1 **b)** 0.05 **c)** 0.025 **d)** 0.01 **e)** 0.005

26. Suppose that a random sample of the volumes in cubic feet of 30 black cherry trees in the Allegheny National Forest has been taken. Assume that the sample mean for the volumes of the 30 trees is $\bar{x} = 30.2$ and the sample variance is $s^2 = 268.96$. If we assume further that the distribution of the volumes of all the black cherry trees is normal, then a confidence interval for the variance σ^2 with confidence coefficient 0.9 is given by

a) (184.3, 436.3) **b)** $(178.2, 421.8)$ **c)** (189.6, 455.7) **d)** (149.0, 594.5) **e)** (183.3, 440.5)

27. Prospective employees of an engineering firm are told that engineers in the firm work at least 45 hours per week, on the average. A random sample of 40 engineers in the firm showed that, for a particular week, they averaged 44 hours of work with a standard deviation of 3 hours. What is the smallest significance level with which we can reject the null hypothesis that the average working time per week is at least 45 hours?

a) 0.9826 **b)** 2.108 **c)** 0.0348 **d)** 0.0174 **e)** 0.4826

28. One-hour carbon monoxide concentrations in air samples from a large city average 10 ppm, with a standard deviation of 9 ppm. Find the probability that the average concentration in 100 samples selected randomly will exceed 12 ppm.

a) 0.4129 **b)** 0.0132 **c)** 0.4868 **d)** 0.0871 **e)** 0.2514

29. The amount of Guinness sold weekly by McGuires Irish pub is normally distributed with mean 2,000 pints and standard deviation of 500 pints. How many pints should Mr. McGuire have on hand at the beginning of the week to be 97.5% sure that he will not run out before the end of the week?

a) $2,044$ **b)** $1,020$ **c)** $2,980$ **d)** 1956 **e)** $4,420$

30. Each time a soccer player takes a penalty, her chances of scoring a goal are 0.7. Let Y be the number of goals she scores out of 5 penalties. What is the standard deviation of Y?

a) $\sqrt{1.05}$ **b)** 1.05 **c)** 3.5 **d)** $\sqrt{3.5}$ **e)** $\sqrt{1.5}$