Name: _____ **Instructor:** Heide Gluesing-Luerssen

Math 214: Introduction to Statistics
Spring Semester 1999
Final Examination
May 4, 1999
4:15 – 6:15 pm

This Examination consists of 29 multiple choice problems worth 5 points each. No partial credit will be given. You start with 5 points. Record your answers by placing an X through one letter for each problem. This booklet consists of 12 sheets of paper including the front cover and one blank page at the end. You are provided with 2 sheets of tables. You may use a calculator and your prepared sheet of notes. The notes and the calculator must not be passed to another student.

Answers to Multiple Choice Problems

1.	(a)	(b)	(c)	(d)	(e)	**16.**	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	**17.**	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	**18.**	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	**19.**	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	**20.**	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	**21.**	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	**22.**	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	**23.**	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	**24.**	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	**25.**	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	**26.**	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	**27.**	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)	**28.**	(a)	(b)	(c)	(d)	
14.	(a)	(b)	(c)	(d)	(e)	**29.**	(a)	(b)	(c)	(d)	
15.	(a)	(b)	(c)	(d)	(e)						

I have not violated the Honor Code in this examination.

Signature: _____

Score: _____

1. 10 applicants are available for 4 identical jobs, 3 of them are male, and 7 are female. A supervisor selects 4 of these applicants randomly. What is the probability that at least 2 males are selected?

(a) $\frac{9}{56}$

(b) $\frac{1}{3}$

(c) $\frac{3}{5}$

(d) $\frac{31}{42}$

(e) $\frac{16}{105}$

2. What is the probability that a random five letter "word" made (with repetitions allowed) from the seven letters $\{ \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G} \}$ has all its letters different? Assume that the letters are chosen randomly, i. e., that any letter chosen from the set $\{ \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G} \}$ is equally likely. (Note: order counts, e. g. ABCDD is a different word than BACDD.)

(a) $\frac{P_5^7}{7^5}$

(b) $\frac{5!}{7!}$

(c) $\frac{P_5^7}{7!}$

(d) $\frac{\binom{7}{5}}{7^5}$

(e) $\frac{1}{P_5^7}$

3. Assume A and B are events having probabilities $P(A) = 0.5$, $P(B) = 0.2$, and $P(AB) = 0.1$. Calculate $P(B \mid \bar{A})$.

(a) 0.8

(b) 0.2

(c) 1

(d) 0.25

(e) 0.4

4. At a certain research institute 50% of the members have cookies in the afternoon. 60% of the members have tea in the afternoon, and 15% of the members have neither. What fraction of the members have both cookies and tea?

(a) 95%

(b) 85%

(c) 75%

(d) 15%

(e) 25%

5. A blood typing test predicts type A for 39% of a given population. 41% of the population actually has type A blood. If a person from the population has type A blood, then the test will predict type A blood for that person with probability 0.88. If the test gives the type of a person's blood as A, what is the probability that the blood is actually type A?

(a) $\frac{0.88 \cdot 0.39}{0.41}$

(b) $0.88 \cdot 0.39$

(c) $0.88 \cdot 0.41$

(d) $\frac{0.41}{0.88}$

(e) $\frac{0.88 \cdot 0.41}{0.39}$

6. More Americans support a ban on handguns today than at any time since 1982, a Newsweek poll released April 24, 1999, said. In the wake of a teenage killing spree in a Colorado high school, 70% of those polled think "stricter gun control would reduce violent crime in the United States". Assuming that this proportion is representative for the U.S. adults, what is the probability that among the next 15 randomly selected adults at most 10 will be of this opinion?

(a) 0.972

(b) 0.722

(c) 0.278

(d) 0.515

(e) 0.485

7. Given a discrete random variable X with probability distribution $\begin{array}{c|cccc} X & -\frac{1}{2} & 0 & 1 & 4 \\ \hline p(x) & \frac{1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \end{array}$.

Find $E(2X+1)$.

(a) 0

(b) 2

(c) 1

(d) 0.5

(e) 3

8. You roll a fair die twice. Denote by X the sum of the upper face numbers and by Y the total number of sixes in both tosses. What is $P(X=8\,|\,Y=1)$?

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{1}{18}$

(d) $\frac{1}{3}$

(e) $\frac{1}{36}$

9. In a certain Starbucks café approximately 15% of the customers order a cappuccino grande. Assuming that customers place their orders independently, what is the probability that on a certain day the 7th customer is the 4th one who orders a cappuccino grande?

(a) $\binom{7}{4}(0.15)^3(0.85)^4$

(b) $(0.15)^4(0.85)^3$

(c) $\binom{7}{4}(0.15)^4(0.85)^3$

(d) $\binom{6}{3}(0.15)^4(0.85)^3$

(e) $\binom{6}{3}(0.15)^3(0.85)^4$

10. The average number of oil tankers arriving each day at a certain port city is found to be 5.6. The facilities at the port can handle at most 7 tankers per day. What is the probability that on a given day tankers have to be turned away, assuming that arrivals follow a Poisson distribution?

(a) 0.797

(b) 0.670

(c) 0.330

(d) 0.203

(e) 0.114

11. Let X be a continuous random variable having probability density function

$$f(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $E(X)$.

(a) 1

(b) 0

(c) $\frac{3}{4}$

(d) $\frac{3}{8}$

(e) $\frac{1}{2}$

12. Let Y be a continuous random variable having probability density function

$$f(y) = \begin{cases} \frac{y}{2} & \text{if } 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $V(Y)$.

(a) $\frac{2}{3}$

(b) 2

(c) $\frac{2}{9}$

(d) 4

(e) $\sqrt{\frac{2}{3}}$

13. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{if } -1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $P(-1 \leq X \leq 1)$.

(a) 0

(b) $\frac{1}{9}$

(c) $\frac{2}{9}$

(d) 2

(e) $\frac{2}{3}$

14. The pH of water samples from a specific lake is a continuous random variable X with $E(X) = 5.5$ and $V(X) = 0.16$. Having no other information about the probability density function of X, which of the following intervals should contain at least $\frac{8}{9}$ of the pH measurements?

(a) (5.1, 5.9)

(b) (4.7, 6.3)

(c) (5, 6)

(d) (4.3, 6.7)

(e) (4.9, 6.1)

15. Suppose that a system contains a certain type of component whose time in years to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\theta = 5$. If four of these components are installed in different systems, what is the probability that all four are still functioning at the end of 5 years?

(a) 0.5

(b) $\frac{1}{16}$

(c) e^{-5}

(d) e^{-4}

(e) $e^{-\frac{4}{5}}$

16. Given the normally distributed variable X with $E(X) = 5$ and $V(X) = 16$. Find $P(1 \leq X \leq 2)$.

(a) 0.4321

(b) 0.6147

(c) 0.0679

(d) 0.7266

(e) 0.9321

17. The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what fraction of the loaves are longer than 31.7 centimeters?

(a) 0.1977

(b) 0.3023

(c) 0.3170

(d) 0.4995

(e) 0.4582

18. A company pays its employees an average wage of \$15.90 an hour with a standard deviation of \$1.50. If the wages are approximately normally distributed and paid to the nearest cent, the highest 10% of the wages is greater than what amount?

(a) \$18.36

(b) \$17.12

(c) \$17.82

(d) \$16.90

(e) \$18.97

19. Let X, Y be random variables with

$$E(XY) = 7,\ E(X) = 2,\ E(Y) = 4,\ V(X) = 4,\ V(Y) = 25.$$

Calculate the correlation coefficient ρ of X and Y.

(a) 0.1

(b) -0.1

(c) 0.39

(d) -0.15

(e) 0

20. Let X and Y be two discrete random variables having the following joint probability distribution.

		X			
		-1	1	2	
	0	0.02	0.03	0.05	
Y	1	0.06	0.09	0.15	
	4	0.12	0.18	0.3	

Exactly one of the following statements is true. Which one is it?

(a) $P(X = 1 \,|\, Y = 4) = 0.6$

(b) $P(Y = 1) = 0.4$

(c) X and Y are independent

(d) $P(X + Y = 0) = 0.02$

(e) $E(Y) = 3$

21. The distribution of heights of a certain breed of poodles has a mean height of 28 cm and a standard deviation of 5 cm. Assuming that the sample mean can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of 36 poodles exceeds 30 cm.

(a) 0.3446

(b) 0.0082

(c) 0.4918

(d) 0.9918

(e) 0.1554

22. Of the customers entering a certain bike store only 20% make purchases. If 100 customers enter the store tomorrow, find the approximate probability that at least 30 make purchases.

(a) 0.0043

(b) 0.0064

(c) 0.4912

(d) 0.2776

(e) 0.0088

23. For a random sample of 25 observations, taken from a normal population with variance $\sigma^2 = 6$, find the probability $P(3.462 \leq S^2 \leq 10.745)$.

(a) 0.995

(b) 0.945

(c) 0.895

(d) 0.94

(e) 0.95

24. A random sample of 12 graduate students of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute, find the 95% confidence interval for the average number of words typed by all graduates of this school.

(a) (74.34, 84.26)

(b) (74.39, 84.21)

(c) (77.53, 81.07)

(d) (75.60, 83.00)

(e) (75.26, 83.34)

25. In a random sample of 1000 homes in a certain city, it is found that 228 are heated by oil. Find the 99% confidence interval for the true proportion of homes in this city that are heated by oil.

(a) (0.202, 0.254)

(b) (0.194, 0.262)

(c) (0.206, 0.25)

(d) (0.215, 0.241)

(e) (0.221, 0.235)

26. The following data, recorded in days, represent the length of time to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication 1	$n_1 = 16$	$\bar{x}_1 = 19$	$s_1^2 = 1.8$
Medication 2	$n_2 = 14$	$\bar{x}_2 = 17$	$s_2^2 = 1.5$

Find the 99% confidence interval for the difference $\mu_1 - \mu_2$ in the mean recovery time for the two medications, assuming normal populations with equal variances.

(a) (0.697, 3.303)

(b) (0.836, 3.164)

(c) (0.785, 3.215)

(d) (0.651, 3.349)

(e) (1.88, 2.12)

27. A marketing expert for a pasta-making company believes that at most 30% of pasta lovers prefer lasagna. It is found that 37 out of 100 pasta lovers choose lasagna over other pastas. Which of the following is the smallest significance level with which we can reject the null hypothesis that at most 30% of pasta lovers prefer lasagna?

(a) 0.437

(b) 1.528

(c) 0.152

(d) 0.126

(e) 0.063

28. A random sample of 50 bags of White Cheddar Popcorn weighed, on average, 5.39 ounces with a standard deviation of 0.51 ounces. Is there enough evidence to conclude that the average weight of a popcorn bag is less than 5.5 ounces? Test the null hypothesis $H_0 : \mu \geq 5.5$ at the significance levels $\alpha = 0.05$ and $\alpha = 0.1$.

(a) Reject H_0 at $\alpha = 0.1$ and don't reject H_0 at $\alpha = 0.05$

(b) Reject H_0 at $\alpha = 0.05$ and don't reject H_0 at $\alpha = 0.1$

(c) Reject H_0 at $\alpha = 0.1$ and at $\alpha = 0.05$

(d) Don't reject H_0 at $\alpha = 0.1$ and don't reject H_0 at $\alpha = 0.05$

29. In the American Heart Association journal *Hypertension*, researchers report that individuals who practice Transcendental Meditation (TM) lower their blood pressure significantly. A random sample of 20 male TM practitioners meditate on average for 8.6 hours per week with a standard deviation of 2.02 hours. Test the null hypothesis that, on average, men who use TM meditate for at most 8 hours at the significance levels $\alpha = 0.05$ and $\alpha = 0.01$. Assume normal distribution for the meditation time per week.

(a) Reject H_0 at $\alpha = 0.05$ and don't reject H_0 at $\alpha = 0.01$

(b) Reject H_0 at $\alpha = 0.01$ and don't reject H_0 at $\alpha = 0.05$

(c) Reject H_0 at $\alpha = 0.01$ and at $\alpha = 0.05$

(d) Don't reject H_0 at $\alpha = 0.01$ and don't reject H_0 at $\alpha = 0.05$