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Mathematics 214: Introduction to Statistics
Review for Exam 2, March 1999

1. Consider a binomial experiment for $n = 20$ and $p = 0.05$. Use the appended table to find $P(Y \geq 3)$. **Write your answer on the cover sheet!**

2. Let Y be a discrete random variable having a Poisson distribution with mean $\lambda = 1$. Use the appended table to find $P(3 \leq Y \leq 5)$. **Write your answer on the cover sheet!**

3. 10% of the nails produced by a certain machine have defects. If 20 nails are randomly selected one at a time for inspection, as the machine produces them, find the variance of the number of inspected nails that have defects.

a) 2 **b)** 1.8 **c)** $\sqrt{1.8}$ **d)** $(1.8)^2$ **e)** 18

4. Let Y denote a random variable having a geometric distribution, with probability of success on any trial given by $p = 0.3$. Find $P(Y = 4)$.

a) $(0.7)^3(0.3)$ **b)** $1 - (0.3)^3(0.7)$ **c)** $1 - (0.7)^3(0.3)$ **d)** $1 - \binom{3}{1}(0.3)(0.7)^3$

e) $1 - 0.3 - (0.7)(0.3) - (0.7)^2(0.3)$

5. Sixty percent of a population of consumers is reputed to prefer Brand A toothpaste. If a group of consumers is interviewed, what is the probability that exactly 5 people must be interviewed before a consumer is encountered who prefers Brand A?

a) $4(0.4)^4(0.6)$ **b)** $\binom{4}{1}(0.4)^4(0.6)$ **c)** $5(0.4)^4(0.6)$ **d)** $(0.6)^4(0.4)$

e) $(0.4)^4(0.6)$

6. 5% of the lightbulbs manufactured on a certain assembly line are defective. If bulbs are randomly selected one at a time and tested, find the probability that the third nondefective lightbulb is found on the fifth trial.

a) $\binom{5}{3}(0.05)^2(0.95)^3$ **b)** $\binom{4}{2}(0.05)^2(0.95)^3$ **c)** $(0.05)^3(0.95)^2$

d) $\binom{4}{2}(0.05)^3(0.95)^2$ **e)** $(0.05)^2(0.95)^3$

7. A certain manufacturer advertises batteries that will run for an average of 150 minutes, with a standard deviation of 10 minutes. Find the smallest interval, which contains at least 75% of the performance periods for batteries of this type?

a) $50 \leq Y \leq 250$ **b)** $140 \leq Y \leq 160$ **c)** $110 \leq Y \leq 190$ **d)** $130 \leq Y \leq 170$

e) $145 \leq Y \leq 155$

8. The number of telephone calls coming into a central switchboard of an office building has a Poisson distribution and averages 7 per minute. Find the probability that at least one call will arrive within a given two minute period.

a) $1 - e^{-1}$ **b)** $1 - e^{-14}$ **c)** $e^{-14} + 14e^{-14}$ **d)** $2(1 - e^{-7})$

e) $(1 - e^{-7})^2$

9. The proportion of time X that a certain computer system is in operation during a week is a random variable with probability density function

$$f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find $E(X)$.

a) 4 **b)** 1 **c)** $\frac{4}{5}$ **d)** $\frac{2}{3}$ **e)** 0

10. For the computer in the last problem, find the probability that the computer is in operation at most half of the week, i. e., $P(X \leq 0.5)$.

a) $\frac{15}{16}$ **b)** $\frac{1}{40}$ **c)** $\frac{1}{4}$ **d)** $\frac{3}{64}$ **e)** $\frac{1}{16}$

11. Let X be a random variable with the uniform distribution

$$f(x) = \begin{cases} \frac{1}{3} & 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

as its probability density function. Find $V(X)$.

a) $\frac{3}{4}$ **b)** $\frac{1}{4}$ **c)** $\frac{3}{2}$ **d)** $\frac{9}{2}$ **e)** 9

12. Telephone calls coming into a certain switchboard follow a Poisson distribution. It is known that during a given 5-minute period, one call arrived at the switchboard. Find the probability that the call arrived within the first minute of this period.

a) $\frac{4}{5}$ **b)** $5e^{-5}$ **c)** $\frac{1}{5}e^{-5}$ **d)** $\frac{1}{4}$ **e)** $\frac{1}{5}$

13. Assume that the number of fatal accidents on scheduled domestic passenger airlines follows a Poisson distribution with a mean of one fatal accident every 41 days. Then the probability distribution for the interaccident times (time between fatal accidents) on scheduled domestic passenger flights is a(n):

a) Poisson distribution **b)** binomial distribution **c)** exponential distribution

d) normal distribution **e)** uniform distribution

14. An engineer has observed that the gap times between vehicles passing a certain point on a highway have an exponential distribution with a mean of 10 seconds. Find the probability that the next gap observed will be no longer than 1 minute.

a) e^{-6} **b)** $1 - e^{-6}$ **c)** $1 - \frac{1}{10}e^{-6}$ **d)** $1 - e^{-60}$ **e)** $1 - e^{-1}$

15. Let the random variable Y possess a probability density function

$$f(y) = \begin{cases} cy & 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find c.

a) $\frac{1}{4}$ **b)** 2 **c)** $\frac{1}{2}$ **d)** 4 **e)** 1

16. Find $P(Z \leq 2)$ where Z is a standard normal random variable. **Write your answer on the cover sheet!**

17. Assume X is a normal random variable with mean 3 and standard deviation 2. Find $P(-1 \leq X \leq 5)$. **Write your answer on the cover sheet!**

18. The grade-point averages of a large population of college students are approximately normally distributed with mean equal to 2.4 and standard deviation equal to 0.5. Suppose that three students are randomly selected from the student body. What is the probability that all three will possess a grade-point average in excess of 3.0?

a) $(0.3849)^3$ **b)** $(0.1151)^3$ **c)** $(0.8849)^3$ **d)** $1 - (0.1151)^3$ **e)** $1 - (0.8849)^3$

1. 0.075

2. 0.079

3. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)

12. (a) (b) (c) (d) (e)

13. (a) (b) (c) (d) (e)

14. (a) (b) (c) (d) (e)

15. (a) (b) (c) (d) (e)

16. 0.9772

17. 0.8185

18. (a) (b) (c) (d) (e)