Peter Cholak Math 221 Wednesday, September 25 Exam 1
This exam is worth 100 points. You should have 7 problems.

(10 points total) Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 1 & 6 \end{bmatrix}$.

(5 points) Find $\det(A)$. Justify your answer.

(5 points) Does A have an inverse? Why?

(30 points total) It is to your advantage to read all of the parts of this problem before beginning. Let

$$A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}.$$

(10 points) Put the matrix A into reduced row echelon form by using *only* elementary row operations. Show all your work and explicitly write out the elementary row operations in the order used. (This can be done using 6 elementary row operations.)

(5 points) For each one of the above elementary row operations, find a matrix E_i such that the matrix $E_i B$ is the matrix that results from doing the ith elementary row operation to the matrix B.

(2 points) A has an inverse. Why?

(3 points) Write A^{-1} as the product of the above elementary matrices.

(3 points) Compute $\det(A)$ and $\det(A^{-1})$. (HINT: compute $\det(A^{-1})$ first.)

(3 points) Solve the system $A^{-1}\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$

(10 points) Suppose that C is a 3×3 matrix. Let E be the appropriate product of elementary matrices so that EC is in reduced row echolon form. In this case, assume that

$$E = \begin{bmatrix} 0 & 3 & -2 \\ 1 & 0 & 0 \\ 0 & 2 & 5 \end{bmatrix} \text{ and } EC = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Form the new 3 by 4 matrix $A = \begin{bmatrix} & & 1 \\ C & & 0 \\ & & 1 \end{bmatrix}$, and solve the system of equations $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$. Be sure to give all of the solutions.

(15 points total) Consider the matrix $B = \begin{bmatrix} 2 & 0 & 2 \\ -1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$.

(10 points) Compute the characteristic polynomial $\det(\lambda I - B)$.

(5 points) For what values of λ does the equation $B\mathbf{x} = \lambda\mathbf{x}$ have non-trivial solutions?

(20 points total) Assume that the polynomial function $f(x)$ has the following form: $f(x) = ax^2 + bx + c$.

(5 points) Write down the system of linear equation that a, b and c, must satisfy in order that the graph of f pass through the points $P = (-1, -1)$, $Q = (1, 1)$, and $R = (2, 0)$.

(15 points) Find all the functions f of the given form which pass through the points P, Q and R?

Decide whether each of the following statements is True or False, and give an explanation.

(7 points) (True/False and why) If A is an invertible matrix, and B and C are matrices so that $BA = CA$, then $B = C$.

(8 points)(True/False and why) The sum of two invertible matrices is again an invertible matrix.