Peter Cholak Math 221 Friday, November 1 Exam 2
This exam is worth 100 points. There are 7 questions on 3 pages (both sides) and one blank page.

(12 points) Let b be a fixed real number. Use *Cramer's Rule* to find the y-coordinate of the solution to the following system of linear equations. No credit will be given for the use of methods other than Cramer's Rule.

$$\begin{bmatrix} -3 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ b \end{bmatrix}$$

(13 points) Find a basis and the dimension of the *solution space* to the equation

$$\begin{bmatrix} 3 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first reflects a vector about the x-axis, and then rotates the vector counter-clockwise by $\frac{3\pi}{4}$ radians (135 degrees).

(9 points) Compute $T(\vec{e}_1)$ and $T(\vec{e}_2)$.

(6 points) Determine the *standard matrix* $[T]$ of the transformation T.

(5 points – no partial credit) Decide which of the following accurately describes the effect of the linear transformation T^{-1} on a vector in $\mathbb{R}^2$.

1. T does not have an inverse.
2. T^{-1} first reflects the vector about the x-axis, and then it rotates the vector counter-clockwise $\frac{3\pi}{4}$ radians.
3. T^{-1} first reflects the vector about the x-axis, and then it rotates the vector clockwise $\frac{3\pi}{4}$ radians.
4. T^{-1} first rotates the vector counter-clockwise $\frac{3\pi}{4}$ radians, and then reflects the vector about the x-axis.

5. T^{-1} first rotates the vector clockwise $\frac{3\pi}{4}$ radians, and then reflects the vector about the x-axis.

Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by the rule $S(x, y, z) = (2x + y - 4z, x + y, 3z)$.

(6 points) Is $(0, \pi, 0)$ an eigenvector for S? If so, what is the corresponding eigenvalue? (10 points) Let $\vec{v} = (2d, d, 1)$, d a real number. Is there a value d

such that $\vec{v}$ is an eigenvector for S? If so, what is the corresponding eigenvalue.

In the following, we specify a vector space V and a subset W of V. In each case, determine if W is a *subspace* of V. Justify your conclusion.

(5 points) $V = \mathrm{Mat}_{2\times 2}$, the vector space of two by two matrices.

W = all two by two matrices A with the property that $A \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(5 points) $V = \mathbb{R}^3$.

W = all vectors $\vec{x} = (x_1, x_2, x_3)$ so that $\vec{x} \cdot (0, 3, -1) = 5$.

Consider the subset B of $\mathbb{R}^4$ consisting of the following three vectors:

$$\vec{b}_1 = (2, 0, 2, 0) \quad \vec{b}_2 = (0, 1, 0, 1) \quad \vec{b}_3 = (0, 1, 0, -1)$$

(10 points) Is B a basis for Span(B)? Why or why not?

(11 points) Decide if $\vec{v} = (1, -1, -1, 2)$ is in Span(B). If so, write $\vec{v}$ as a linear combination of b_1, b_2, b_3. If not, is $\{\vec{u}, \vec{b}_1, \vec{b}_2, \vec{b}_3\}$ a basis for $\mathbb{R}^4$?

Decide whether the following assertion is true or false. Explain your decision.

(8 points) Let $\vec{v}$ be a vector in $\mathbb{R}^3$. If $\vec{v}$ is orthogonal to $\vec{e}_2$, then $\vec{v}$ is in $\mathrm{Span}(\vec{e}_1, \vec{e}_3)$.