Peter Cholak Math 221 Wednesday, December 4, 1996 Exam 3

This exam is worth 100 points. There are 5 problems on 6 pages and one blank page. Be sure to show all your work; you will not get full credit unless your work matches your answer!

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 4 & 1 \\ 1 & 0 & -1 \end{bmatrix}$. (5 points) Find a basis for the row space of A. (6 points) Find a basis for the column space of A. (6 points) Find a basis for the nullspace of A. (3 points) Determine the value of rank(A) and nullity(A).

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 4 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ (the same matrix as in the last problem). (Hint: consider the column space of A.) (6 points) Determine whether $A\vec{x} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ is consistent and if so, find the general solution. (6 points) Determine whether

$A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$ is consistent and if so, find the general solution.

(8 points) Determine the value(s) of b such that $A\vec{x} = \begin{bmatrix} 0 \\ b \\ 2 \end{bmatrix}$ is consistent and find the general solution for this value(s).

For each of the following, indicate whether the statement is true or false. Include a brief explanation of your answer. (You will not full credit without a reasonable explanation!).

(8 points) Let V be an inner product space, and let $S = \{\vec{v}_1, \vec{v}_2\}$ be an *orthogonal* set of vectors. Then S is a linearly independent set of vectors.

(6 points) Let A be a 3×7 then the general solution to the system of linear equations $A\vec{x} = \vec{b}$ has at least 4 parameters. (6 points) Let S be a orthonormal

basis for an inner product space V, $(\vec{u})_S = (1, 2, 0)$ and $(\vec{v})_S = (-1, 2, 1)$. Then the distance between $\vec{u}$ and $\vec{v}$ is 4, i.e. $d(\vec{u}, \vec{v}) = 4$.

(20 points) Use the Gram-Schmidt process to find an *orthogonal* basis for the row space of A, where

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 3 & 0 & 1 \end{bmatrix}$$

Let W denote the span of the following vectors in $\mathbb{R}^4$:

$$\begin{array}{lcl} \vec{b}_1 & = & (\ \tfrac{4}{5}, \ 0, \ \tfrac{3}{5}, \ 0 \ ) \\ \vec{b}_2 & = & (\ -\tfrac{3}{5}, \ 0, \ \tfrac{4}{5}, \ 0 \ ) \\ \vec{b}_2 & = & (\ 0, \ 1, \ 0, \ 1 \ ) \end{array}$$

(4 points) Verify that $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ is an orthogonal basis for W. (9 points)

Let $\vec{x} = (1, 2, 1, 0)$. Compute $\mathrm{Proj}_W(\vec{x})$, the *orthogonal projection* of $\vec{x}$ onto the subspace W. (7 points) Find an orthonormal basis for $W^\perp$. (Hint: first think

about what the dimension of $W^\perp$ should be and then think about what you did in the last part.)