Math 221

Some Practice Problems for Exam 2 – Math 221

The second exam will cover sections 2.4, 4.1,4.2,4.3,5.1,5.2,5.3, and 5.4. The following problems were taken from last year exams. If anything there are not enough problems from sections 5.3 and 5.4 listed here – do the ones in the book which were not assigned. The problems are not listed in order.

1. Find all eigenvalues and eigenvectors

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

2. Find x_2 by **Cramer's** rule (no credit will be given if other method is used).

$$\begin{array}{ccccccc} x_1 & & & + & x_3 & = & 3 \\ x_1 & - & x_2 & - & x_3 & = & -1 \\ & & x_2 & + & x_3 & = & 4 \end{array}$$

3. Write down the cofactor expansion along the *second* row. Do not evaluate.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix}$$

5. Multiple choice questions.

(i). Let $\mathbf{u}, \mathbf{v}$ be two vectors in R^n. If $\|\mathbf{u}\| = 2$ and $\|\mathbf{w}\| = 3$, then

(A). $|\mathbf{u} \cdot \mathbf{w}| \leq 6$, (B). $|\mathbf{u} \cdot \mathbf{w}| = 6$, (C). $|\mathbf{u} \cdot \mathbf{w}| \geq 6$, (D). none of the above.

(ii). Suppose that A is a $n \times n$ matrix such that $\det A = 0$. Let $T_A : R^n \to R^n$ be a linear operator with its standard matrix A. Which of the following statements is true?

(A). The range of T_A is R^n, (B). T_A is one to one, (C). T_A is invertible, (D). none of the above.

7. Determine which sets are vector spaces under the given operations.

(i). The set of all vectors of the form $(x, y, 1)$ with usual addition and scalar multiplication.

(ii). The set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ with matrix addition and scalar multiplication.

(iii). The set of all real positive functions with usual addition and scalar multiplication.

(iv). The set of all real-valued functions f defined everywhere on the real line and such that $f(1) = 2$, with usual addition and scalar multiplication.

8. Find two vectors in R^2 with Euclidean norm 1 whose Euclidean inner product with $(3, -1)$ is zero.

9. Find the standard matrix for the linear operator defined by the formula $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 4x_1 - 5x_2)$.

10. Find the standard matrix for the stated composition on R^2:

(a). A counterclockwise rotation by $30°$, followed by a reflection about the line $x = y$.

(b). An orthogonal projection on the x-axis, followed by a contraction with factor $k = \frac{1}{2}$.

11. Let $T : R^2 \to R^2$ be a linear operator given by the formula $T(x_1, x_2) = (x_1 - x_2, x_1 + 2x_2)$. Find a formula for the inverse operator $T^{-1}(w_1, w_2)$.

12. Determine which of the following sets of vectors are linearly dependent.

(a), x, x^2, x^3.

(b), $(0, 0, 0)$, $(1, 1, 1)$, $(-1, 0, 3)$.