Math 221

Some Practice Problems for Exam 3 – Math 221

The exam will cover everything since the last exam: 5.5,5.6,6.1,6.2,6.3,6.5. We did NOT cover the QR-dcomposition in 6.3. There will be somewhere between 20 and 30 points on multi-guess problems on this exam. Since this has put together quickly the answers and numbers used need not be nice.

Here are some problems from the text you should be able to solve: 5.5: 5,6,8,9,10,12; 5.6: 4,5,6,14; 6.1: 9,16; 6.2: 8,9,12,16,18-21 (good multi-guess type problems); 6.3: 13,19,21; 6.5: 10,22.

Consider the following bases of R^2:

$\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v}_1 = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix}$.

Express $\vec{u}_1 = a\vec{v}_1 + b\vec{v}_2$. (HINT: is $\vec{v}_1, \vec{v}_2$ a nice basis?)

Express $\vec{u}_2 = c\vec{v}_1 + d\vec{v}_2$.

Let $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$. Let $\vec{x} = 3\vec{v}_1 + 2\vec{v}_2$. Use the matrix A to express $\vec{x}$ as a linear combination of $\vec{u}_1$ and $\vec{u}_2$.

Consider the matrix $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$. Assume that A is row equivilent to the matrix $R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

Find a basis for the column space of A.

Find a basis for the row space of A.

Is the vector $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ in the null space of A? Answer "yes", "no", or "can't tell". Explain your answer.

Is the vector $\begin{bmatrix} 2 \\ \sqrt{5} \end{bmatrix}$ in the column space of A? Answer "yes", "no", or "can't tell". Explain your answer.

Let S be a basis for an inner product space V. Let $\vec{u}$ and $\vec{v}$ be vectors in V such that $(\vec{u})_S = (1, 2, 3, 4)$ and $(\vec{v})_S = (-4, 3, -2, 1)$. Answer the following: What is the dimension of V? The norm of $\vec{u}$? of $\vec{u}$? Are $\vec{u}$ and $\vec{v}$ orthogonal? Compute the following: the norm of $4\vec{u} - 2\vec{v}$, $< 2\vec{u}, -3\vec{v} - \vec{u} >$. Find the coordinates of a vector which is orthogonal to both $\vec{u}$ and $\vec{v}$.

Using Gram-Schmidt process to transform the following basis to an orthonormal basis.

$\mathbf{u_1} = (0, 0, 1), \mathbf{u_2} = (1, 1, 2), \mathbf{u_3} = (1, 2, 2)$

Let $M = \begin{bmatrix} 1 & -1 & 3 & 0 \\ 2 & -1 & 3 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

Find a basis for the null space of M

Show that the vector $\vec{y} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ is in the column space of M.

(ii). Suppose that A is a 4×6 matrix, the system $A\mathbf{x} = \mathbf{b}$ is consistent, and Rank$(A) = 3$. Then the number of parameters in the general solution is

(A). 1, (B). 2, (C). 3, (D). 4.

(iii). If A is a matrix with more rows than columns, then

(A). Dim(row space)<Dim(column space),

(B). Dim(row space)>Dim(column space),

(C). Dim(row space)=Dim(column space).

(iv). If $A = (\mathbf{c_1}, \mathbf{c_2}, \mathbf{c_3}, \mathbf{c_4})$ is a 5×4 matrix and $\mathbf{b} = \mathbf{c_1} - \mathbf{2c_2} + \mathbf{3c_3} - \mathbf{4c_4}$, then

(A). the system $A\mathbf{x} = \mathbf{b}$ is consistant,

(B). the system $A\mathbf{x} = \mathbf{b}$ is not consistant,

(C). the consistancy of the system $A\mathbf{x} = \mathbf{b}$ depends on $\mathbf{b}$.

Determine which of the following are inner product. Circle your answer.

(i). the space R^3, $< \mathbf{u}, \mathbf{v} >= u_2v_2 + u_3v_3$, (here $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$), (Yes, No).

(ii). the space R^3, $< \mathbf{u}, \mathbf{v} >= u_1v_1 + 2u_2v_2 + 3u_3v_3$, (here $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$), (Yes, No).

(iii). the space R^3, $< \mathbf{u}, \mathbf{v} >= u_1^2v_1^2 + u_2v_2 + u_3^2v_3^2$, (here $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$), (Yes, No).

(iv). the space R^3, $< \mathbf{u}, \mathbf{v} >= u_1v_1 + u_2v_2 - u_3v_3$, (here $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$), (Yes, No).

Suppose that $< \mathbf{u}, \mathbf{v} >= 1, \|\mathbf{u}\| = 2, \|\mathbf{v}\| = 3$ Find $\|\mathbf{u} + \mathbf{v}\|$.

Let W be the line in R^2 with equation $y = 5x$. Find the equation for $W^\perp$.