Peter Cholak Math 221

Here are some practice problems from Chapter 7. The exam will have at least one problem from sections 7.1 and 7.2.

Our exam will be in this room on Wednesday, December 18 from 8 am. to 10 am.

Consider the 3×3 matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & -1 \end{bmatrix}$. Find the eigenvalues of A. The matrix A is diagonalizable. Why? Compute P so that $P^{-1}AP$ is a diagonal matrix. Find this diagonal matrix. Compute A^5 (do this the "easy way" using the matrix P from the previous part of the problem). (Hint: use elementary operations to find A^{-1} and to multiply the matrices.)

Let $S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the eigenvalues of S. Find 3 linearly independent eigenvectors for S. Normalize these eigenvectors. Find P so that $P^{-1}SP$ is diagonal. Show that you can choose P to be an *orthogonal* matrix. Compute A^6 (do this the "easy way" using the matrix P from the previous part of the problem). Note since P is orthogonal $P^{-1} = P^T$ so no computation is needed to find P^{-1}. Why? And again you can use elementary operations to multiply the matrices.)

Determine whether the following matrixes are diagonalizable or not. Explain why or why not! Do not diagonalize! $\begin{bmatrix} 2 & 0 & 3 & 3 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. $\begin{bmatrix} 2 & 0 & 3 & 3 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. $\begin{bmatrix} 2 & 1 & 3 & 3 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.