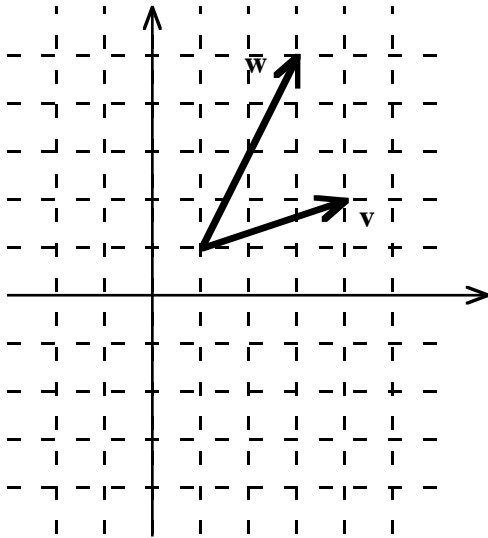


Student's name

1.(5 points)



In the diagram on the left draw the vector $\mathbf{v} - \frac{1}{2}\mathbf{w}$.

Assuming that the grid represents a unit of measure find from the picture the components of the vectors

\mathbf{v} , \mathbf{w} , $\mathbf{v} - \frac{1}{2}\mathbf{w}$.

2. (10 points) Find the angle between the two vectors \mathbf{v} and \mathbf{w} above.

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3. (5 points) The matrices A and B are invertible with inverses A^{-1} and B^{-1} . What is the inverse of the block matrix $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$?

4. (15 points) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}$. Find the factorization $A = LDU$, where L is a lower triangular matrix with 1-s on the diagonal, D is a diagonal matrix and U is an upper triangular matrix with 1-s on the diagonal.

Change the entry $A(3, 3)$ in such a way that such a factorization becomes impossible.

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5. (15 points) Represent the elements of the column space of the matrix A

$= \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ as linear combinations of the smallest number of columns.

Use this representation to determine for which of the following vectors, if any, the equation $A\mathbf{x} = \mathbf{b}$ has a solution:

a) $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, b) $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Find a c , such that the vector $\begin{bmatrix} 2 \\ 1 \\ c \end{bmatrix}$ belong to the columns space of A above.

6. (10 points) Compute the square of the matrix $\begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}$. Decide without further computation what is its inverse.

7. (5 points) Which of the following matrices is in reduced echelon form?

$$A = \begin{bmatrix} 1 & 0 & -5 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. (10 points) For one that is find the nullspace.

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9. (10 points) Define the following:

- (a) The transpose of a matrix A

- (b) A symmetric matrix

- (c) A skew symmetric matrix

10. (15 points) True or false?

- (a) For any three unit vectors \mathbf{x} , \mathbf{y} , \mathbf{z} , if $\mathbf{x} \cdot \mathbf{y} = 0$ and $\mathbf{x} \cdot \mathbf{z} = 0$, then $\mathbf{y} = \mathbf{z}$ or $\mathbf{y} = -\mathbf{z}$.

- (b) $(AB)^{-1} = A^{-1}B^{-1}$.

- (c) The number of rows in the echelon form and the reduced echelon form of a matrix are always the same.

- (d) Every square matrix is a sum of a symmetric matrix and a skew symmetric matrix.

- (e) The inverse of a symmetric matrix is symmetric.

(f) A 3×3 skew symmetric matrix has no inverse.