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1. The vectors (1, 2, 3) and (3, 2, 1) are two particular solutions of a non homogeneous equation $A\mathbf{x} = \mathbf{b}$, where A is a matrix of rank 2.

   (a) Find a basis for the nullspace of the matrix A?

   (b) Find the general solution.

   (c) Explain why the information that the rank of A is 2 is essential?

2. What is the dimension of the subspace of $\mathbf{R}^4$, which is spanned by the vectors

   (0, 0, 0, 0), (1, 1, 1, 1), (1, 2, 3, 4), (4, 3, 2, 1)?
   Find a basis for this subspace.

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3. A vector space V is spanned by 5 vectors. Next to the following statements mark "yes" if the statement is certainly true, "maybe" if the statement can be true but not necessarily, "no" if the statement is never true.

(A) The dimension of V is 5. ❑ yes, ❑ maybe, ❑ no.

(B) The dimension of V is 3. ❑ yes, ❑ maybe, ❑ no.

(C) The dimension of V is less or equal 5. ❑ yes, ❑ maybe, ❑ no.

(D) There are 6 independent vectors in V. ❑ yes, ❑ maybe, ❑ no.

(E) There are 3 independent vectors in V. ❑ yes, ❑ maybe, ❑ no.

(F) Every set of vectors that spans V contains no more than 5 vectors.
❑ yes, ❑ maybe, ❑ no.

Do the same for the following statements without the restriction on the size of the matrix.

(G) If the dimensions of the nullspace and the left nullspace of a matrix A are equal , then A is a square matrix. ❑ yes, ❑ maybe, ❑ no.

(E) If A is a skew symmetric matrix, then the column space and row space of A coincide. ❑ yes, ❑ maybe, ❑ no.

(F) The nullspace and left nullspace of a symmetric matrix coincide.
❑ yes, ❑ maybe, ❑ no.

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4. A subspace of $\mathbf{R}^3$ is defined by the equation $(1, 2, 3)\cdot\mathbf{x} = 0$. Which of the following sets of vectors span the subspace? Which is a basis? Justify your answer.

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

5. Find the dimension and a basis for the four spaces: the row space, the column space, the nullspace and the left nullspace of the matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 6 & 0 \end{bmatrix}$.

6. The vectors (1, 0, 1, 0), (0, 1, 0, 1) and (1, 1, 0, 0) span a three dimensional subspace of $\mathbf{R}^4$. Use the Gramm-Schmidt orthogonalization process to find an orthonormal basis for this subspace. Find a fourth vector, which, together with the three you found, forms an orthonormal basis for $\mathbf{R}^4$

Extra credit questions 5 points each:

Define: 1. Independent vectors

2. Basis of a vector space.