1.(20 points) Use Cramer's formulas to solve the following system of equations

$$\begin{aligned} x - y \quad\;\; &= 3 \\ y + z &= 3 \\ x - y + z &= 2 \end{aligned}$$

Warning: No credit will be given for any other method of solution.

2. (15 points). A sequence of matrices is defined as follows:

$$M_1 = [\,1\,],\ M_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},\ M_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \dots,$$

$$M_n =$$

$$\begin{bmatrix} 1 & 2 & \dots & n \\ n+1 & n+2 & \dots & 2n \\ 2n+1 & 2n+2 & \dots & 3n \\ \dots & \dots & \dots & \dots \\ (n-1)n+1 & (n-1)n+2 & \dots & (n-1)n+n \end{bmatrix}, \dots$$

Show that $\det M_n = 0$ for all $n \geq 3$. Why are the first two different?

3. (15 points) Use the big formula to evaluate the determinant $\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \end{vmatrix}$.

Write out explicitly all nonzero products with the proper sign.

4. (20 points) $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$. What are the eigenvalues of A, A^2 and A^3 ?

Find an eigenvector corresponding to the eigenvalue 3 of A.

Is this an eigenvector of A^2 ? If so, to which eigenvalue of A^2 does it belong.

Are these matrices diagonalizable? Justify.

5. (15 points) The characteristic polynomial of the matrix $\begin{bmatrix} 8 & 3 & -3 \\ -6 & -1 & 3 \\ 12 & 6 & -4 \end{bmatrix}$ factors into $-(\lambda+1)(\lambda-2)^2$. Decide if the matrix is. diagonalizable?

6. (20 points) The matrices $A = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ have the same eigenvalues as the matrix in question 5. Which of these matrices is diagonalizable?

7. Extra credit. The eigenvalues of the matrix $A = \begin{bmatrix} 23 & -36 \\ -36 & 2 \end{bmatrix}$ are -25 and 50. Find the orthogonal matrix, which diagonalizes A.