Math 221 – McNinch
November 17, 1997
Additional Problems

1. Consider the matrix $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$

(a). Compute the **characteristic polynomial** $det(A - \lambda I)$. This will be a third degree polynomial in λ.

(b). Compute $A \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Deduce that $\lambda_1 = 1$ is an eigenvalue of A.

(c). Find all eigenvalues of A. (Try dividing the characteristic polynomial by $(\lambda - 1)$. Why would this be useful?)

(d). Find the eigenvectors corresponding to the eigenvalues you just found.

(e). If we write the characteristic polynomial as

$$c_0 - c_1\lambda + c_2\lambda^2 - \lambda^3,$$

check that $c_0 = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$, where the λ_i are the three eigenvalues you have found. Check that $c_0 = \det(A)$.

(f). Check that $c_2 = \lambda_1 + \lambda_2 + \lambda_3$ and that $c_2 = \mathrm{Tr}(A)$.

(g). In general, c_1 will be given by the formula $\lambda_1 \cdot \lambda_2 + \lambda_2 \cdot \lambda_3 + \lambda_1 \cdot \lambda_3$. Check that this formula gives the correct answer in this case. Show that this is always the correct formula when B is a triangular matrix $\begin{bmatrix} \lambda_1 & a & b \\ 0 & \lambda_2 & c \\ 0 & 0 & \lambda_3 \end{bmatrix}$.

(h). Can A be diagonalized to a diagonal matrix Λ? Find Λ.