Name: ____________________

September 29, 1997

Exam 1

Show your work in completing the following problems. There are 6 pages on the exam. Remember to pace yourself; don't spend too much time on a single problem. Good luck!

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Points per problem:
     Problem 1. 8 points            Problem 2. 8 points
     Problem 3. 5 points            Problem 4. 5 points
     Problem 5. 10 points           Problem 6. 24 points
     Problem 7. 22 points           Problem 8. 18 points
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1. Give complete definitions for the following terms. Use complete sentences. [4 pts each]

(a). *Invertible matrix:*

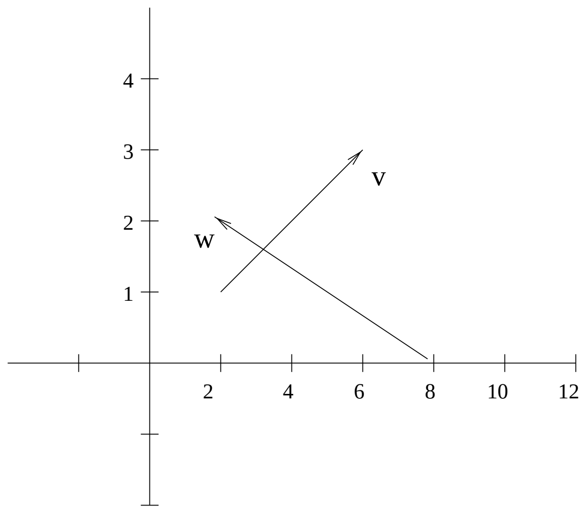
(b). *Column space of a matrix:*

2. True/False. If false, demonstrate with an example. [4 pts each]

(a). Let A be a 2×2 matrix. If the equation $A\vec{x} = \vec{b}$ has a solution for every $\vec{b} \in \mathbb{R}^2$, then A is invertible.

(b). If the matrices A and B are symmetric, then the *block* matrix $\begin{bmatrix} \vec{0} & A \\ B & \vec{0} \end{bmatrix}$ is symmetric.

3. [5 pts] Consider the following vectors in $\mathbb{R}^2$:



Sketch and give the components of the vector difference $\vec{v} - \vec{w}$:

$$\vec{v} - \vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

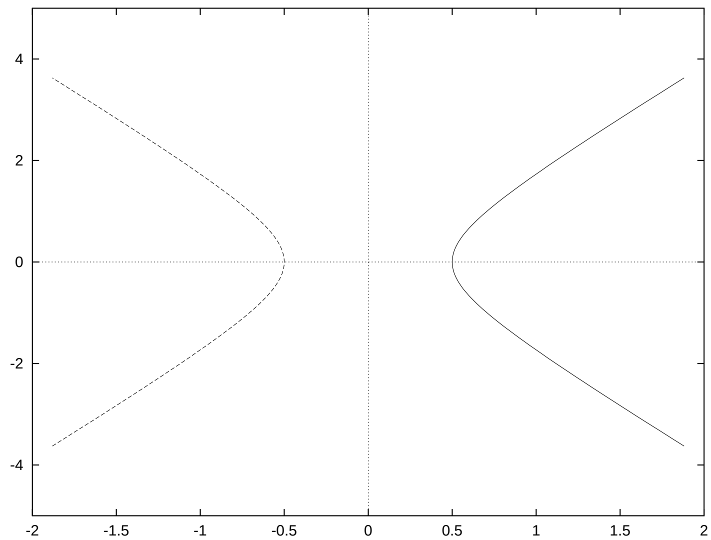
4. [5 pts] Consider the plane defined by the equation: $\vec{n} \cdot \vec{v} = 3$ where $\vec{n} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

Find the scalar c so that $c \cdot \begin{bmatrix} 1 \\ 300 \\ -2 \end{bmatrix}$ lies on the plane.

5. [10 pts] The hyperbola $ax^2 - by^2 = c$ passes through the points

$$(x, y) = (0.5, 0), \text{ and}$$

$$(x, y) = (0.707, 1)$$



Find a coefficient matrix A and a vector (c_1, c_2) so that the coefficients a, b, c satisfy the equation

$$A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

Don't solve your system!

6. [12 pts each] Find the inverses of the following matrices.

(a). $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

(b). $A = \begin{bmatrix} 1 & -3 & -a \\ 0 & 3 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

7.

(a). [12 pts] Write $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ as a product $B = U^T \cdot D \cdot U$, where U is an upper triangular matrices *with 1's on its diagonal,* and D is a diagonal matrix.

(b). [10 pts] Find a 2×2 matrix A so that $A_{2,2} = 1$, but $E_{2,1}A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

8. Consider the matrix $A = \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a). [6 pts] (Circle the appropriate choices.)

- The column space of A is a subspace of ($\mathbb{R}^2$, $\mathbb{R}^3$).
- The column space of A is (the zero subspace, a line, a plane, some other space).

(b). [6 pts] Does the equation $A\vec{x} = \vec{b}$ have a solution when $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$?

Solve the equation for $\vec{x}$ if there is a solution.

(c). [6 pts] Does the equation $A\vec{x} = \vec{b}$ have a solution when $\vec{b} = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$?

Solve the equation for $\vec{x}$ if there is a solution.