Name: ______________________
October 29, 1997

Exam 2

Show your work in completing the following problems. There are 6 pages on the exam. Remember to pace yourself; don't spend too much time on a single problem. Good luck!

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Points per problem:
     Problem 1. 12 points              Problem 2.  17 points
     Problem 3. 17 points              Problem 4.  18 points
     Problem 5. 18 points              Problem 6.  18 points
                    Extra Credit: 4 points
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1. Give definitions for the following terms. Remember to write complete, coherent sentences.

(a). *Linearly independent vectors:*

(b). *Null space of an* $m \times n$ *matrix* A*:*

2. Find an equation $aX + bY + cZ + dW = 0$ which describes the “plane” in $\mathbb{R}^4$ spanned by the vectors:

$$\begin{aligned} v_1 = &\ (1, 0, \tfrac{1}{3}, 0) \\ v_2 = &\ (1, 2, 1, 0) \\ v_3 = &\ (1, 0, 1, \tfrac{1}{2}) \end{aligned}$$

3. Let α and β be **fixed** real numbers. Construct a 2×4 matrix A whose null space is the span of the following two vectors.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ \alpha \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ -1 \\ \beta \\ 3 \end{bmatrix}$$

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & 3 & -3 & 1 \\ 1 & 1 & 5 & -1 & 1 \end{bmatrix}$$

(a). Find a basis for the *null space* of A (i.e. for $N(A)$).

(b). What is the *dimension* of $N(A)$?

(c). Find a basis for the *column space* of A.

(d). What is the *rank* of A?

(e). What is the dimension of the *left null space* of A?

5. Consider the following set of *matrices* regarded as vectors.

$$\vec{m}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \\ 1 & 0 \end{bmatrix}, \vec{m}_2 = \begin{bmatrix} 0 & 1 \\ 1/2 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and } \vec{m}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}.$$

(a). Decide whether the set $\{\vec{m}_1, \vec{m}_2, \vec{m}_3\}$ is linearly independent. **Justify** your answer.

(b). Decide whether the matrix $\begin{bmatrix} 10 & 10 \\ 5 & 5 \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$ is in the span of $\{\vec{m}_1, \vec{m}_2, \vec{m}_3\}$. Again, justify your answer.

6. Decide if the following statements are **true** or **false**, and give a brief explanation or example (whichever is appropriate).

(a). If A is a 20×30 matrix, there are always at least 10 linearly independent vectors in the null space of A.

(b). The rows of a 4×5 matrix are always linearly dependent.

(c). The set $\{(a,b) : a \geq b\}$ is a subspace of $\mathbb{R}^2$.

7.

EXTRA CREDIT: (4 POINTS)

Let $i = \sqrt{-1}$. Find a complex number $z = a + ib$ so that $\vec{v} = \begin{bmatrix} i \\ z \end{bmatrix}$ is in the null space of the matrix

$$A = \begin{bmatrix} 1+i & 2 \\ 1 & 1-i \end{bmatrix}$$