Name: ______________________________
November 24, 1997

Exam 3

Show your work in completing the following problems. There are 6 pages on the exam. Remember to pace yourself; don't spend too much time on a single problem. Good luck!

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Points per problem:
     Problem 1. 10  points             Problem 2.  12  points
     Problem 3. 20  points             Problem 4.  20  points
     Problem 5. 18  points             Problem 6.  20  points
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1. Give definitions for each of the following terms. Use *complete and coherent* sentences!

1. An orthonormal basis.

2. The (i, j) cofactor of an $m \times m$ matrix A, for $1 \leq i \leq m$ and $1 \leq j \leq m$.

2. Construct the following matrices A, or tell why no such matrix exists.

(a). A 3×3 matrix A with exactly 2 eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and with the dimension of the null space of $A - I = A - \lambda_1 I$ equal to 1.

(b). A 3×3 matrix A with only the single eigenvalue $\lambda = 2$ and with $\det(A) = 1$.

3.

(a). Use the Gram-Schmidt process to find a matrix Q with orthonormal columns which has the same column space as the matrix A, where

$$A = \begin{bmatrix} 3 & 4 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

(b). Using Q find the vector $\vec{x}$ in the column space of A closest to $\vec{v} = (1,-1,1,-1)$. Is $\vec{v}$ in the column space of A?

The vector $\vec{x} =$ __________________ In column space? __________________

4. Let $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$.

(a). Find matrices S and Λ so that $S^{-1}AS = \Lambda$ is a diagonal matrix.

(b). Solve the equation $A^{10}\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

5. Let $\vec{v} = \frac{1}{\sqrt{3}}(1,-1,1)$ and let $Q = I - 2\vec{v}\cdot\vec{v}^T$, where I is the 3×3 identity matrix.

(a). Show that Q is an orthogonal matrix.

(b). Solve the equation $Q\vec{x} = (1,1,1)$.

6. Consider the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Use Cramer's rule to answer the following.

(a). Find the y coordinate of the solution to equation (1):

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix} \qquad \text{where } \alpha \text{ is any scalar.} \tag{1}$$

(b). Find the y coordinate of the solution to equation (2):

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \tag{2}$$

(c). Explain any similarities or differences between the previous two answers. In which coordinate (x, y, or z) do the solutions to (1) and (2) differ? In which coordinate are the solutions the same?