Name: ____________________________
December 8, 1997

Review for Final Exam

The final exam is comprehensive. It will cover chapters 1-6 and sections 7.1 and 7.2.

1. The final exam will be held on December 18, at 7:30 pm. It will be in deBartolo, but *not in the usual class location.* Instead, it will be in room 136 deBartolo.

2. The final exam will be common between the two sections of Math 221. I believe that the final exams for the two sections are not at the same time. Thus, let me remind you not to discuss the contents of the exam with other students who might thus get an advantage.

3. Study the old Practice Exams and Exams. These are all now on the course web site (www.nd.edu/ gmcn-inch/math221). Know especially the following things:

1. How to use row operations to put a matrix in echelon form, and use this reduction to solve matrix equations $A\vec{x} = \vec{b}$.
2. Know the properties of matrix operations (addition, multiplication). Know what an inverse matrix *is.* Know how to find a matrix inverse using row reduction.
3. Remember how to factorize a matrix $A = L \cdot D \cdot U$ where L is lower triangular, U is an echelon matrix, L and U have 1's on their diagonals, and D is a diagonal matrix.w
4. Know what a subspace is.
5. Know how to find the column space, null space, row space of a matrix.
6. Know how to find a particular solution to an equation $A\vec{x} = \vec{b}$, and know how to use the special solutions in the null space to give the general solution to the matrix equation.
7. Understand linear independence and spanning. Be able to identify and find bases for subspaces. Know what the dimension of a vector space is.
8. Know the "fundamental theorem of linear algebra" relating the dimensions of the four subspaces.
9. Understand *planes* in $\mathbb{R}^3$ given by equations $aX + bY + cZ = 0$, given by equations $\vec{v} \cdot \vec{n} = 0$, and given by spanning sets: $a\vec{v}_1 + b\vec{v}_2$.
10. Know what orthogonal subspaces are. Remember that the null space is the orthogonal complement to the row space.
11. Know what an orthogonal basis, orthonormal basis, orthogonal matrix are. Know how to use Gram Schmidt to find orthonormal bases for subspaces starting with arbitrary bases for subspaces.
12. Know how to find the projection of a vector onto a line. Know how to find the projection of a vector onto a subspace. Remember that if $q_1, \ldots, q_m$ is an orthonormal basis for a subspace, then the projection of v onto the subspace is easy to find; it is just $p = (q_1^T v)q_1 + \cdots + (q_m^T v)q_m$. In matrix form, this says that $p = QQ^T v$.

13. Know what the defining properties of the determinant are (see §5.1). Know how to compute determinants using co-factor expansion.
14. Know Cramer's rule, and know how to obtain the inverse of a matrix from the co-factor matrix.
15. Know what an eigenvalue and an eigenvector are. Know how to diagonalize a matrix. Remember the theorem stating that any symmetric real matrix can be diagonalize by an orthogonal matrix.
16. Know how to compute the characteristic polynomial of a matrix. Know how the null space of the matrix $A - \lambda I$ relates to the eigenvectors with eigenvalue λ.
17. Know what a linear transformation is. Know how to find the matrix of a linear transformation

You should work examples of the above types of problems, in order to make sure that you understand the material. In class on Wednesday, I will come up with examples of some of these problems as I am asked. Or, you can ask me to work problems from the old practice exams or the homework assigned throughout the term.

4. Do the following problems: §7.2: 1, 3, 5, 6, 7, 15.