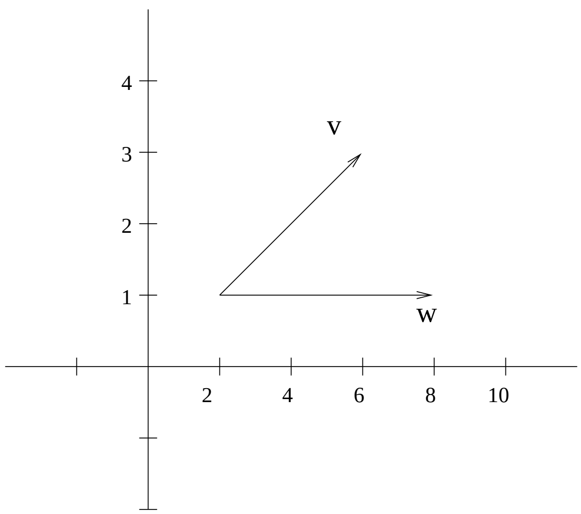
Math 221 – Section 02
Prof. George McNinch
Name: ______________________
September 24, 1997

Practice Exam 1

1. Consider the following vectors $\vec{v}$ and $\vec{w}$.



(a). What are the components of $\vec{v}$ and $\vec{w}$?

$$\vec{v} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

(b). Compute the lengths $|\vec{v}|$ and $|\vec{w}|$.

(c). Compute the angle θ between $\vec{v}$ and $\vec{w}$.

(d). On the give graph, sketch the vector difference $\vec{v} - \vec{w}$ and the difference $\vec{w} - \vec{v}$.

2. Let $\vec{w} = (3, 1, 3, -1)$.

(a). Find two *unit vectors* in the direction of $\vec{w}$.

(b). Find two different vectors perpendicular to $\vec{w}$.

3. Consider the plane defined by the equation:

$$(X - 1) + 2(Y + 1) = 3(Z - 2)$$

(a). Does the plane pass through the origin? If not, find a vector $\vec{v}_0$ which does lie on the plane.

(b). Find the *normal vector* $\vec{n}$ to the plane. Describe the plane by an equation of the form $\vec{n} \cdot \vec{v} = d$. (i.e. tell me what d is!)

(c). What multiple $c\vec{n}$ of the normal vector lies on the plane? Find the distance between the plane and the origin $\vec{0} = (0, 0, 0)$.

4. The parabola $x = ay^2 + b$ passes through the points $(x, y) = (-1, 2)$ and $(x, y) = (1, 1)$

(a). Give a matrix equation $A \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ which the coefficients a and b must satisfy.

(b). Solve your system. Give the equation of the parabola.

5. Find the inverses of the following matrices.

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}, \qquad \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & a \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

6.

(a). Write $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}$ as a product $A = D \cdot P$ where D is a *diagonal* matrix, and P is a *permutation* matrix.

(b). Write $B = \begin{bmatrix} 0 & 0 & 3 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ as a product $B = P \cdot D$ where D is a *diagonal* matrix, and P is a *permutation* matrix.

7. For each of the following matrices, give the factorization $A = L \cdot D \cdot U$, where L and U are lower and upper triangular matrices *with 1's on their diagonals,* and D is a diagonal matrix.

(a). $A = \begin{bmatrix} 3 & 9 & 27 \\ 0 & 2 & 24 \\ 0 & 0 & 1 \end{bmatrix}$. (*Hint:* the matrix L doesn't have to be interesting!)

(b). $A = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 7 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. (*Hint:* notice that A is symmetric!!)

8. In general, two elimination steps are needed to make $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & b & 1 \end{bmatrix}$ into an upper triangular matrix. Give the elimination matrices $E_{2,1}$ and $E_{3,2}$ which perform these steps. Find a single matrix B so that $B \cdot A$ is upper triangular. Check your answer by computing $B \cdot A$.

9. Find the column space $R(A)$ for the following matrices. For the given vectors $\vec{b}$, decide if the system $A\vec{x} = \vec{b}$ has a solution.

(a). $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix}$ $\vec{b} = (1,1,0)$, $\vec{b} = (1,0,1)$.

(b). $A = \begin{bmatrix} -1 & 3 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\vec{b} = (1,1)$, $\vec{b} = (1,-3,0)$, $\vec{b} = (2,1,1)$.

10. True/False. If false, demonstrate with an example.

(a). Given any two non-zero vectors $\vec{v}$ and $\vec{u}$, there is always a linear combination $a\vec{v} + b\vec{u}$ which is perpendicular to $\vec{v}$.

(b). For any $m \times n$ matrix A, the matrix $A^T A$ is a symmetric matrix.

(c). If A and B are square matrices, then $(A \cdot B)^T = A^T \cdot B^T$.

(d). If the matrices A and B are invertible, then the *block* matrix $\begin{bmatrix} \vec{0} & A \\ B & \vec{0} \end{bmatrix}$ is invertible. (You might think about how to find the inverse.)

11. Give definitions of the following terms:

(a). Symmetric matrix

(b). Permutation matrix

(c). Inverse matrix

(d). the Transpose of a matrix

(e). orthogonal vectors