Exam 1 is *Friday, October 13.* It will cover Chapter 1 (sections 1.1 - 1.8), Chapter 2 (sections 2.1 - 2.3), Chapter 3 (sections 3.1 - 3.3) and the first third sections of Chapter 4. A good source of practice problems for the exam can be found in the homework problems assigned (especially the uncollected ones, if you didn't do them yet...). The quizes are also good place to look.

The exam will have some definitions, some true/false with reasons problems, some computations problems and one or two short proof-like problems.

Prof. Cholak (office: 223 CCMB) will have drop in office hours during the day on Thursday from 9 am to 11 am and 1 pm to 5 pm. Prof Wong (office: 234 CCMB) will have drop in office Thrusday from 8 pm to 11 pm.

Make sure you know the definitions of the terms used in the class. Here are some that you might keep in mind (this list might not include everything that you need to know!). Note that it is often very useful to have in mind some examples; as this is a useful way to help you remember terminology, you might think about what examples seem most natural to you.

- What does it mean for a linear system to be consistent?
- What is an elementary row operation?
- What are row echelon and reduced row echelon matrices?
- What is a pivot position in a matrix? pivot column?
- What is a free variable? general solution?
- What is a linear combination? What is the *span* of a set of vectors? What does it mean for some vectors to *span* R^n?
- If $\vec{v}$ and $\vec{w}$ are vectors in R^n, what is $\vec{v} + \vec{w}$? What is $c\vec{v}$ for a scalar c?
- What is a homogeneous system?
- What is linearly independence?
- What is a linear tranformation? If T is a linear transformation, what is meant by the *domain* of T? the *range* of T? the *codomain* of T? What does it mean to say that T is one-to-one? that T is onto?
- What is the standard matrix of a linear transformation?
- If A and B are matrices, when can (and how does...) one define $A + B$? $A \cdot B$? rA where r is a scalar? What is A^T?
- What does it mean to say that a matrix A is invertible?
- What is an elementary matrix?
- If A is an $n \times n$ matrix and $1 \leq i, j \leq n$, what is meant by the notation $A_{i,j}$? $C_{i,j}$? what is a co-factor?

- Give the formula for $\det A$ using cofactor expansion along the ith row, where $1 \leq i \leq n$ is fixed. Then give the definition using cofactor expansion along the jth column, where $1 \leq j \leq n$ is fixed.

- You should review the definition of a vector space. I'm not likely to ask you to list all 10 axioms, but you should certainly be familiar with them.

- Define a *subspace* of a vector space V.
- Let A be an $m \times n$ matrix. Define the null space $Nul(A)$. What vector space is the null space a subspace of? Define the column space $Col(A)$. What vector space is the column space a subspace of?
- define the *kernel* of a linear transformation $T : V \to W$, where V and W are vector spaces.
- define a *basis* for a vector space V.
- what is the *standard basis* for R^n? for P_n?

Much of the first part of this course involves rephrasing problems in various ways. You should know how to translate! For example:

- Given a system of linear equations, you should know how to produce the corresponding *vector equation* and *matrix equation* (also various permuations of this: e.g. instead *start* with a matrix equation...)
- Be able to explain why a matrix equation $A\vec{x} = \vec{b}$ is consistent $\iff$ $\vec{b}$ is in the span of the columns of A $\iff$ the last column of the corresponding augemented matrix is not a pivot column.
- Explain why the vectors $\vec{v}_1, \ldots, \vec{v}_m$ are linearly independent if and only if the homogeneous matrix equation $A\vec{x} = \vec{0}$ has only the trivial solution, where $A = [\vec{v}_1 \ldots \vec{v}_m]$ is the matrix whose columns are the $\vec{v}_i$.
- Let T be a linear transformation $T : R^n \to R^m$, and let A be its standard matrix. (What size matrix is A?) Explain why T is one-to-one if and only if the columns of A are linearly independent. Explain why T is onto if and only if the columns of A span R^m.

True/False: (as usual, if the statement is true give a reason, otherwise provide an example showing that it is false.) Another good place to look is at the True/False in the book.

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors in the vector space V, and let $H = Span(\vec{u}, \vec{v}, \vec{w})$. Suppose that $\vec{u} + 2\vec{v} + 3\vec{w} = \vec{0}$. Then $\{\vec{u}, \vec{v}\}$ is a basis for H. Let A and B be $m \times m$ matrices. Then $\det(A + B) = \det(A) + \det(B)$. The subset $\{1aa + b : a, brealnumbers\}$ of R^3 is a *subspace*. The set $\{\cos(\theta), \sin(\theta)\}$ form a linearly independent subset of the vector space of all continuous functions from R to R. Suppose $\mathcal{S} = \{\vec{v_1}, \ldots, \vec{v_n}\}$ spans the vector space V. Then some subset of $\mathcal{S}$ is a basis for V. Let A be an $m \times n$ matrix. If $Nul(A) = \{\vec{0}\}$, then the columns of A form a basis for the column space $Col(A)$.

Consider the equations x - y + z - w = 0
z - 2w = 0 Describe the solution set S as the span of a collection of vectors in R^4. Let $A = 1 - 11 - 1$
$001 - 2$, and let $\vec{b_1} = 4$
2, $\vec{b_2} = 17$

2. Give general solutions to the equations $A\vec{x} = \vec{b_1}$ and $A\vec{x} = \vec{b_2}$. Use your answers to describe *all* possible matrices C with the property that $AC =$ 174
22.

Give examples as directed; be sure to explain why your example does what you claim. On the exam, you *will not* get full credit if you don't explain your examples! Give an example of a linear transformation $T : R^2 \to R^3$ which is one-to-one but not onto. Give an example of a linear transformation $T : R^3 \to R^2$ which is onto but not one-to-one.

Give an example of a linearly dependent set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ in R^4 such that each of the subsets $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, $\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$, $\{\vec{v}_1, \vec{v}_3, \vec{v}_4\}$, and $\{\vec{v}_2, \vec{v}_3, \vec{v}_4\}$ are linearly independent. Give an example of a plane in R^3 that contains neither the vector $\vec{e_1} = 100^T$ nor the vector $\vec{e_2} = 010^T$.

True/False questions. If the statement is true, give a reason. If it is false, provide an example which shows that it is false. Suppose that A and B are $n \times n$ matrices, and that $AB = \vec{0}$. Then A is singular (i.e. A is not invertible).

If A is an invertible $n \times n$ matrix, then $A + I_n$ is also invertible.

If A is an $m \times n$ matrix whose columns do not span R^n, then the equation $A\vec{x} = \vec{b}$ is inconsistent for every $\vec{b}$ in R^n.

Let A be a 20×30 matrix. Suppose that $A\vec{x} = \vec{e_{19}}$ is a consistent equation. ($\vec{e_i}$ denotes the column whose ith entry is 1, and whose other entries are 0.) Then $A\vec{x} = \vec{e_{19}}$ has infinitely many solutions.

A 2×10 matrix always has at least 2 pivots.

Let $I = I_2$ be the 2×2 identity matrix. Let K be a 2×2 matrix satisfying $K^2 = -I$. Show that K is invertible. Let $J = 0 - 1$
10. Show that $J^2 = -I$. Let P be an invertible 2×2 matrix. Show that $K = PJP^{-1}$ satisfies $K^2 = -I$. Show that there are infinitely many 2×2 matrices K such that $K^2 = -I$. (use part (c)!)

Find the inverses of the following matrices. $A = 2000$
0300
$00 - 10$
$000\frac{1}{4} \cdot 0100$
1000
0001
0010

$A = 1 - 3 - a$
$03 - 6$
001 A=$\cos(\theta)\sin(\theta)$
$-\sin(\theta)\cos(\theta),\qquad 0 \leq \theta < 2\pi.$

Consider the matrix $A = -1 - 2$
10
01

Does the equation $A\vec{x} = \vec{b}$ have a solution when $\vec{b} = 1$
2
2?

Does the equation $A\vec{x} = \vec{b}$ have a solution when $\vec{b} = -4$
-2
3?

Compute the following determinants:

- $\det(A)$ where $A = a_1 00$
  $2a_2 0$
  $12a_3$

- det 102
  113
  108.

Find an elementary 3×3 matrix E so that $\det(E^2) = \sqrt{3}$.

Let $A = 010$
101
010. For what scalars c is the matrix $cI_3 - A$ an invertible matrix?

Let $A = 1012$
0130
0001. Find a basis for $Nul(A)$. Find a basis for $Col(A)$.

Find a matrix A so that its column space $Col(A)$ is as follows: Col(A) = $\{a + b + c2a - c3b + 4c : a, b, crealnumbers\}$

Let V be a vector space, and let $\vec{u}$ be a non-zero vector in V. If $c\vec{u} = \vec{0}$, prove that $c = 0$.

Consider the function T:$\mathrm{P}_2 \to R^2$ given by the rule $T(\mathbf{p}(t)) = \mathbf{p}(0)$
$\mathbf{p}(1)$ Show that T is a linear transformation. Find a basis for the kernel of T. (what vector space is the kernel a subspace of ??) Show that 1
0 and 0
1 are in the range of T. Using this, describe the range of T.

Let H be the column space of A where = 1201
0131
0010. Find a basis $\mathcal{B}$ for H.

Let $\mathcal{B} = \{11, 1 - 2\}$. Show that $\mathcal{B}$ is a basis for R^2