[time,mathptm]article
Solutions to Practice Problems Exam 1
document
The question are not guaranteed to be correct.
(3) (True/False questions)
a. False. An example is obtained by taking $\vec{u} = \vec{v}$ and $\vec{w} = -\vec{v}$.
b. False. An example is: $A = B = I$ (identity matrix).
c. False. The set doesn't contain the zero vector.
d. True. $\cos(t)$ is not a multiple of $\sin(t)$ (nor vice versa).
e. True. Any spanning set contains a linearly independent spanning set.
f. True. $Nul(A) = 0$ means that the columns are linearly independent.
(4) a. $H = \text{Span}(bmatrix1$
b. The solutions are $H_1 = bmatrix2$
(5) a. $T(bmatrixa$
b. $T(bmatrixa$
c. $S = bmatrix1$
.
d. $\vec{e}_3 + bmatrix1$
(6) a. False. $I0 = 0$.
b. False $-I + I$ is not invertible.
c. True.
d. True.
f. False. $A = 0$.
(7) a. $-KK = I$. So $(-K)K = I$. Hence by the invertible matrix theorem $K$ is invertible.
b. Just multiple it out.
c. $KK = PJP^{-1}PJP^{-1} = PJ^2P^{-1} = P(-I)P^{-1} = -PP^{-1} = -I$.
d. There are infinitely many invertible matrices $P$.
(8) a. $bmatrix01300$
b. $bmatrix119 - a$
c. $bmatrix\cos(\theta) - \sin(\theta)$
(9) a. No
b. Yes $x_1 = -2, x_2 = 3$.
(10) a. $a_1a_2a_3$.
b. 6.
(11) $E = bmatrix\sqrt{[}4]300$
(12) $\det(cI - A) = c^3 - 2c$. So $cI - A$ is invertible unless $c = 0, \sqrt(2), -\sqrt(2)$.
(13) a. Note that $A$ is row equivalent to the reduced echelon matrix $B = bmatrix1010$
b. The pivots in $B$ occur in the 1st, 2nd and 4th columns. Thus columns $1, 2, 4$ of $A$ are pivot columns, hence those columns are a basis.
(14) $Abmatrix111$
(15) Let $\vec{u}$ be a non-zero vector in $V$, and let $c$ be a scalar. Assume that $c\vec{u} = \vec{0}$. If $\neq 0$, then we may speak of the number $1c = c^{-1}$. Moreover, using properties of vector spaces, we see that

$$\vec{0} = c\vec{u}$$

$$c^{-1}(\vec{0} = c\vec{u})$$

$$0 = \vec{u}$$

But $\vec{u}$ is not $\vec{0}$, so the only possibility is that in fact $c = 0$.
(16) a. $T(g(t)) = bmatrixg(0)$
b. $f \in Ker(T)$ iff $f(0) = 0$ and $f(1) = 0$. Hence $f(t) = ct(t-1)$. So $f(t) = t(t-1) = t^2 - t$ is a basis.
c. $T(1-t) = bmatrix1$
(17) Observe that A is row equiv to the following reduced echelon matrix $bmatrix100 - 1$
(18) Since the two elements of B are not multiples of one another, they are linearly independent. They span $R^2$ and hence they form a basis of $R^2$.