Provide the precise definition of each of the following concept: The span of a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.

A set of vectors $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ being linearly independent.

The elementary row operations.

The determinant of an $n \times n$ matrix A.

TRUE/FALSE. Determine whether the followings statements are true or false. Be sure to provide a reason for your answer. Let $T : \mathbf{R}^m \to \mathbf{R}^m$ be a linear transformation. Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a set of linearly independent vectors in $\mathbf{R}^m$. Then the set of vectors $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_n)\}$ is also linearly independent.

Let A and B be two invertible matrices. Then ABA^{-1} is invertible.

If A and B are square and $AB = I$ then $\det(A) = 0$.

Let $T(\mathbf{x})$ be a linear transformation. T is one to one if and only if the kernel of T is $\{0\}$.

Let A be the following matrix:

$$A = abcdefghi$$

if $\det A = 1$ then (hint: use the properties of determinant) $\det abc$
$-2d - 2e - 2f$
$ghi = ?$

$\det abc$
$a + db + ec + f$
$ghi = ?$

Describe the null space of the matrix: $A = 1 - 11 - 1$
$- 22 - 13$
$3 - 231$
$4 - 44 - 4$. Find all solutions of the equation $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = -2$
8
4
$- 8$.

Write 1
4 as a linear combination of 1
1 and -1
2.

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation. If $T(1,1) = (2,3), T(-1,2) = (4,5)$ then $T(1,4) =?$.

Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be linearly independent vectors. Show that $\mathbf{v}_1, \mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3$ are linearly independent vectors.