(4 points each.) Provide the precise definition of each of the following concept: The *basis* of a vector space V.

A *finite-dimensional* vector space V.

The *eigenvalues* of a matrix A.

Two matrices A and B are *similar*.

TRUE/FALSE. Determine whether the followings statements are true or false. Be sure to provide a reason for your answer. (2 points each for correct answer; 2 points each for correct reason.) A is invertible iff $\lambda = 0$ is not a eigenvalue of A.

Let A be a $m \times n$ matrix. Then the column space of A plus the nullspace of A^T is n.

The dimension of P^3 (all polynomials of degree 3 or less) is 3.

All diagonalizable matrices are invertible.

(20 points.) Let $A = 1021$
0101
1122
. Row reduce A.

Find a basis for the column space of A. What the dimension of the column space A?

Find a basis for the row space of A. What the dimension of the row space A?

Find a basis for the null space of A. What the dimension of the null space A?

(20 points.) Let $A = 2d1$
022
003
.

For what values of d is A diagonalizable? For any such value, diagonalize A (do not find P^{-1}).

(15 points.) Consider the vector space $V = \mathbf{M}_2$ of 2×2 matrices. Let $\mathrm{B} = \{E_{1,1} = 1000, E_{1,2} = 0100, E_{2,1} = 0010, E_{2,2} = 0001\}$.

From the sample exam (problem #12) we know $\mathcal{B}$ is a basis for V.

Let $T : V \to V$ be given by the rule $T(M) = M - M^T$. Show that T is a linear transformation. Compute the matrix N of T relative of the basis $\mathcal{B}$. (As in problem #12 N is a 4×4 matrix, not a 2×2 matrix.)

(15 points.) Let A be a 5×8 matrix whose rank is 5. Show that $A\vec{x} = \vec{b}$ is consistent and has infinitely many solutions for all $\vec{b}$ in R^5.