The Final exam will is at *7:30-9:30 pm, Wednesday, December 13 in 117 DBRT*. It is comprehensive and covers the following sections of Chapters 1-6: 1.1-1.8, 2.1-2.3, 3.1-3.3, 4.1-4.7, 5.1-5.4, 6.1-6.5.

You should use the practice exams for review material for the first 5 chapters, below you will find some problems for chapter 6. The last quiz is also a good place to look for problems for chapter 6.

The final will be about 1 and a half times longer than the past exams. As before there will be definitions, True/False problems, some short proofs and some computation problems. There will be more True/False problems but this time you do not have to give your reasoning.

Definitions/terminology from recent material.

If u, v are vectors in R^n, what is their *inner product* or *dot product*, $v \cdot u$? How does one compute $\|v\|$? What is the distance between u and v? Be familiar with the "algebraic" properties of inner products (they are listed in your text; e.g. $u \cdot (v_1 + v_2) = u \cdot v_1 + u \cdot v_2$.) What is meant by an orthogonal set of vectors? an orthonormal set of vectors? What does it mean to say that a vector v is orthogonal to the subspace W? What is meant by an orthogonal basis for a subspace W? Define the orthogonal projection $Proj_W(v) = \hat{v}$ of a vector v onto a subspace W. Be able to give a formula for this projection when an orthogonal basis $u_1, u_2, \cdots u_m$ of W is given. Understand how to carry out the Gram-Schmidt process of constructing an orthogonal basis. Understand the so-called QR decomposition of a matrix A. If the columns of A are independent, the Gram-Schmidt process yields an orthogonal (and even orthonormal) basis whose vectors form the columns of a matrix Q with $Q^T Q = I_m$ (what is m?) Remember that $R = Q^T A$ is upper triangular (this is a good check that you have carried out Gram-Schmidt correctly). If U is an $m \times n$ matrix with orthonormal columns, review the properties of U: e.g. $U^T U = I_n$, $\|Uv\| = \|v\|$ for all vectors v in R^n, $Uv \cdot Uw = v \cdot w$. U is said to be an orthogonal matrix.

True-False questions: If $\{v_1, v_2, v_3\}$ is an orthogonal set and if c_1, c_2, c_3 are scalars, then $\{c_1 v_1, c_2 v_2, c_3 v_3\}$ is an orthogonal set. If $\|u - v\|^2 = \|u\|^2 + \|v\|^2$, then u and v are orthogonal. Every orthogonal set in R^n is linearly independent. For an $m \times n$ matrix A, vectors in the $NulA$ are orthogonal to vectors in $ColA$. If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $x \mapsto Ax$ preserves lengths. Let $W = Span\{v_1, \cdots, v_p\}$. Then x is in $W^\perp$ if and only if x is orthogonal to every v_i, $1 \le i \le p$. If W is a subspace of R^n then the intersection of W and $W^\perp$ is empty. If $y = z_1 + z_2$, where z_1 is in a subspace W and z_2 is in $W^\perp$, then z_1 must be the orthogonal projection of y onto W. If a square matrix has orthonormal columns, then it also has orthonormal rows. If an $n \times n$ matrix P has orthogonal columns, then $P^T = P^{-1}$.

If U is $m \times n$ with orthogonal columns, then $UU^T x$ is the orthogonal projection of x onto $ColU$. The orthogonal projection of y onto u is a scalar multiple of y. If the orthogonal projection of a vector y onto a subspace W is zero, then y is in $W^{\perp}$.

Let U be an $n \times n$ orthogonal matrix. Show that if $\{v_1, \cdots, v_n\}$ is an orthonormal basis for R^n, then so is $\{Uv_1, \cdots, Uv_n\}$.

Show that if an $n \times n$ matrix U satisfies $(Ux) \cdot (Uy) = x \cdot y$ for all x and y in R^n, then U is an orthogonal matrix.

Given a vector v, show that the set $\{x \in R^n \mid x \cdot v = 0\}$ form a subspace of R^n.

Let V be the subspace of R^3 spanned by the vectors $\mathrm{v}_1 = 1$
1
1, $v_2 = 1$
-2
2, $v_3 = 1$
-5
3. Find the dimension of the subspace V of R^3 spanned by the vectors. Find a basis for the orthogonal space $V^{\perp}$ of V. For what values of $c = c_1$
c_2
c_3 does the following system of equations have a solution? x 1
1
1
+ y 1
-2
2 + z 1
-5
3 = c_1
c_2
c_3 If the equation is consistent, how many solutions are there?

Consider the vectors $\mathbf{x_1} = 1$
2
$1, \mathbf{x_2} = 1$
1
0, and $\mathbf{x_3} = 0$
2
2. Let $W = Span\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}\}$. Compute $\dim W$, and find a basis for W. Use the Gram-Schmidt process to find an *orthogonal* basis for the subspace W.

Let $\mathbf{v_1} = 1$
1
0
0, $\mathbf{v_2} = 1$
-1
1
1 and $\mathbf{v_3} = 0$
0
1

$-$ 1.

Show that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is an orthogonal set.

Let $W = Span\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$. Compute $\dim W$.

Let $\mathbf{x} = -1$

$-$ 1

2

$-$ 2. Compute the orthogonal projection $\hat{\mathbf{x}} = Proj_W(\mathbf{x})$. Let V be the subspace of R^3 spanned by the vectors $\mathrm{v}_1 = 1$

1

1, $\mathbf{v_2} = 1$

2

2, $\mathbf{v_3} = 1$

3

3. Find a basis for the orthogonal space $V^\perp$ of V.

Consider the vector equation $x1$

1

$1 + y1$

2

$2 + z1$

3

$3 = c_1$

c_2

c_3. For what values of $\mathbf{c} = c_1$

c_2

c_3 does the system have a solution $\mathbf{x} = x$

y

z?

If the above equation has at least one solution $\mathbf{x}$, how many solutions does it have?

The best quadratic function $f(x) = c + dx + ex^2$ to fit the points $(-1, 1), (0, 1), (1, 0)$ and $(2, 2)$ is obtained using the formula $A^T A\hat{x} = A^T\vec{b}$. Find the matrix A and the vector $\vec{b}$. Solve for $\hat{x}$.

Find an orthonormal basis for $C(A)$ where -166

$3 - 83$

$1 - 26$

$1 - 4 - 3$. Find the QR decomposition of A.

Find all the least square solutions where $A = 12$

24

$-1 - 2$

and $b = 3$

2

1.