Consider the following system of linear equations:

$$2\,x_1 + 4x_2 + 6x_3 = 6$$
$$\mathrm{x}_1 + 3x_2 + 4x_3 = 4$$
$$\mathrm{x}_2 + ax_3 = 1$$

(2 points) Find A and $\vec{b}$ such that the above system in the form $A\vec{x} = \vec{b}$ (i.e. an matrix equation).

$A = 246$
134
$01a$
and $\vec{b} = 6$
4
1
.

(5 points) Explicitly showing *all* the steps row reduce the *augmented matrix* for the above system into echelon form.

The augmented matrix is $M = 2466$
1344
$01a1$
. First lets divide the top row by 2 to get: 1233
1344
$01a1$
. Now multiple the top row by -1 and add to the second row to get 1233
0111
$01a1$
. Subtract the second row from the third row to get $N = 1233$
0111
$00a - 10$
. This is now in echelon form.

(2 points) For which a does the augmented matrix have 3 pivots?

When $a \neq 1$.

(4 points) Explicitly showing *all* the steps reduce the augmented matrix into reduce echelon form. (Hint: there are 2 cases.) First lets assume $a \neq 1$. Then we can divide the last row by $a - 1 \neq 0$ to get: 1233
0111
0010
. Then subtract 3 times the last row from the first row and the last row from the second row to get: 1203
0101
0010
. Now subtract 2 times the second from the first row to get: 1001
0101
0010
which is in reduce echelon form.

Now assume $a = 1$. Hence $N = 1233$

0111
0000
. Subtract two times the second from the first row to get 1011
0111
0000
which is in reduce echelon form. (2 points) Is there an a such that the system does not have a solution? Why or why not?

No! It is never the case that the last column contains a pivot.

(5 points) Find the general solution to the above system (Hint: there are two cases).

There are two cases: either the reduce echelon from is 1001
0101
0010
or 1011
0111
0000
. In the first case, the general solution is $x_1 = 0, x_2 = 1$ and $x_2 = 1$ or $\vec{x} = 1$
1
0
. In the second case, the general solution is $x_1 = 1 - x_3, x_2 = 1 - x_3$ and x_3 is free. Another way to write this is: $\vec{x} = 1 - x_3$
$1 - x_3$
x_3
$= 1$
1
0
$+ -x_3$
$- x_3$
x_3
$= 1$
1
0
$+ x_3 - 1$
$- 1$
1
.