Express the solutions of the equation $A\mathbf{x} = \mathbf{0}$ in parametric form, where

$$A = 1102 - 2 - 21 - 511 - 1344 - 19$$

The reduce row echolon form of A is 1102
$001 - 1$
0000
0000. So x_1 and x_3 are basic variables and x_2 and x_4 are free variables. Hence the general solution is $x_2 - 1$
1
0
$0 + x_4 - 2$
0
1
1.

For what values of c are the vectors $\mathbf{v}_1 = -1$
0
$-1, \mathbf{v}_2 = 2$
1
2 and $\mathbf{v}_3 = 1$
1
c (a) linearly independent? (b) linearly dependent?

We want to consider when the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{v}_3$ has solutions. $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{v}_3$ has solutions iff $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly dependent. For this we need to rowreduce the matrix -121
011
$-12c$. Subtract the first row from the third to get -121
011
$00c - 1$. $c = 1$ iff the last column does not has pivot iff vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{v}_3$ has solutions iff $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly dependent.

Suppose that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are 3 linearly independent vectors show that the vectors $\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$ are also linearly independent.

$c_1\mathbf{v}_1 + c_2(\mathbf{v}_1 + \mathbf{v}_2), (\mathbf{v}_1 + c_3\mathbf{v}_2 + \mathbf{v}_3) = \mathbf{0}$ iff $(c_1 + c_2 + c_3)\mathbf{v}_1 + (c_2 + c_3)\mathbf{v}_2 + c_3\mathbf{v}_3) = \mathbf{0}$. Hence $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is linearly independent iff $\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$ are linearly independent.

Let S be a set of vectors and T be a subset of S. Are the following statements TRUE or FALSE. Be sure to a proof or a counterexample. If the vectors in S are linearly independent then the vectors in T are also linearly independent. If the vectors in S are linearly dependent then the vectors in T are also linearly dependent.

(a) True. If there is a non-trivial linear combination of the vectors in T whose sum is the zero vector then the same is true for S. So if T is linearly dependent then so is S.

(b) False. $T = \{1$
0

$0, 0$
1
$0, 0$
0
$1\}$ and $S = T \cup \{1$
1
$1\}$. T is linearly independent but S is not.

Is the absolute value function $T : \mathbf{R} \to \mathbf{R}, T(x) = |x|$ a linear transformation? Justify your answer.

No. For example, $3 = |-3| \neq -1|3| = -3$.