November 17, 2004

MATH 221 Linear Algebra Quiz IV

1. Let $V_n = \{a_0 + a_1 t + a_2 t^2 + ... + a_n t^n \,|\, a_i \in \mathbf{R}, i = 0, ..., n\}$ be the vector space of polynomials of degree at most n. The set $\{1, t, t^2, ..., t^n\}$ is a basis for V_n and is referred to as the standard basis. Consider the case $n = 3$ and show that:

(**a**) the set $\mathcal{B} = \{1, (1-t), (1-t)^2, (1-t)^3\}$ is also a basis of V_3;

(**b**) what are the coordinates of the polynomial $1 + t + t^2 + t^3$ relative to the standard basis?

(**c**) a polynomial P has coordinates $[1, 1, 1, 1]_{\mathcal{B}}$ relative to $\mathcal{B}$ what are the coordinates of P relative to the standard basis?

(**d**) what are the coordinates of the polynomial $1 + t + t^2 + t^3$ relative to the basis $\mathcal{B}$ given in (**a**)?

(**e**) the set $\mathcal{C} = \{1, (1+t), (1+t)^2, (1+t)^3\}$ is also a basis for V_3 what is the coordinates relative to $\mathcal{C}$ of the polynomial $P = [1, 1, 1, 1]_{\mathcal{B}}$ of part (c)?

2. Given a polynomial $P(t) = a_0 + a_1 t + a_2 t^2 + ... + a_n t^n$ of degree n then its derivative $P'(t) = a_1 + 2a_2 t + ... + n a_n t^{n-1}$ is a polynomial of degree at most $n-1$. Consider the case $n = 3$ and show that:

(**a**) the map $T : V_3 \to V_2$ defined by $T(P(t)) = P'(t)$ is a linear transformation;

(**b**) write down the matrix A representating the transformation T relative to the standard bases for V_3 and V_2?

(**c**) what is the null space of the matrix A in (**b**)?

(**d**) what is the column space of the matrix A in (**b**)?

(**e**) verify the Rank Theorem by using your answers for (**c**) and (**d**).