MATH 221 Linear Algebra Quiz V

(I) Decide which of the following statements, concerning an $m \times n$ matrix A, are true and which are false (no proof is needed however make sure that you understand your answers).

(1) If B is a matrix in echelon form obtained from A via elementary row operations then the row space of A and the row space of B are the same.

(2) If B is a matrix in echelon form then the non-zero rows form a basis for the row space of B.

(3) If B is a matrix in echelon form then the dimension of the row space of B is equal to the number of pivots of B.

(4) If B is a matrix in echelon form then the dimension of the null space of B is equal to the number of free variables in the equation $B\mathbf{x} = 0$.

(5) If B is a matrix in echelon form then the dimension of the row space of B is equal to the number of leading variables in the equation $B\mathbf{x} = 0$.

(6) If B is a matrix in echelon form then the columns of B containing the pivots form a basis of the column space of B.

(7) If B is a matrix in echelon form obtained from A via elementary row operations then whenever certain columns of B form a basis of the column space of B, the corresponding columns of A is a basis of the column space of A.

(8) The dimension of the row space of A is the same as the dimension of the column space of A.

(9) The sum of the dimension of the row space of A and the dimension of the null space of A is n.

(10) The sum of the dimension of the row space of A and the dimension of the null space of A is m.

(11) The sum of the dimension of the column space of A and the dimension of the null space of A is n.

(12) The sum of the dimension of the column space of A and the dimension of the null space of A is m.

(13) The sum of the dimension of the row space of A and the dimension of the column space of A is n.

(14) The sum of the dimension of the row space of A and the dimension of the column space of A is m.

(15) The rank of A is the dimension of the row space of A.

(16) The rank of A is the dimension of the column space of A.

(17) The sum of the rank of A and the dimension of the row space is n.

(18) The sum of the rank of A and the dimension of the row space is m.

(19) The sum of the rank of A and the dimension of the null space is n.

(20) The sum of the rank of A and the dimension of the null space is m.

(II) Let A be an $n \times n$ matrix which of the following statements are NOT equivalent to the staement that A is invertible?

(1) dim Null Space$(A) = 0$

(2) dim Null Space$(A) = n$

(3) rank$(A) = n$

(4) rank$(A) = 0$

(5) dim Column Space$(A) = 0$

(6) dim Columnl Space$(A) = n$

(7) dim row Space$(A) = 0$

(8) dim row Space$(A) = n$

(III) Find the dimension and basis for the null space, column space and row space of A given below. What is the rank of A?

$$A = \begin{pmatrix} 3 & 2 & -4 & 1 & 5 \\ 6 & 4 & -7 & 3 & 1 \\ -3 & -2 & 6 & 1 & 2 \\ 9 & 6 & -11 & 4 & 6 \end{pmatrix}$$