November 17, 2004

MATH 221 Solution to Linear Algebra Quiz V

(I) (1) T, (2) T, (3) T, (4) T, (5) T, (6) T, (7) T, (8) T, (9) T, (10) F, (11) T, (12) F, (13) F, (14) F, (15) T, (16) T, (17) F, (18) F, (19) T, (20) F.

(II) (2), (4), (5), (7)

(III) Reduce A to echelon form, resulting in

$$B = \begin{pmatrix} 3 & 2 & -4 & 1 & 5 \\ 0 & 0 & 1 & 1 & -9 \\ 0 & 0 & 0 & 0 & 25 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

It is clear that

$$\dim RS(A) = \dim CS(A) = rank(A) = 3.$$

By Rank Theorem $\dim CS(A) + \dim NS(A) = n = 5$ hence $\dim NS(A) = 2$.

The non-zero rows of B form a basis for $RS(B) = RS(A)$ so a basis of $RS(A)$ is given by

$$(3, 2, -4, 1, 5),\ (0, 0, 1, 1, -9),\ (0, 0, 0, 0, 25).$$

The pivot columns are the first, the third and the 5th hence the following vectors form a basis of $CS(A)$:

$$\begin{pmatrix} 3 \\ 6 \\ -3 \\ 9 \end{pmatrix}, \begin{pmatrix} -4 \\ -7 \\ 6 \\ -11 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 2 \\ 6 \end{pmatrix}.$$

To find the null space we further reduce the matrix B to reduced echelon form

$$C = \begin{pmatrix} 1 & 2/3 & 0 & 5/3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since $NS(A) = NS(B) = NS(C)$ the null space consists of sloutions of the equation $C\mathbf{x} = 0$. The free variables are x_2 and x_4 hence the solutions are given by

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -\frac{3}{2} \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -\frac{5}{2} \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$