Math 221 – Linear Algebra
Quiz 1

1. (10 points) Consider the following system of linear equations:

$$
\begin{array}{rcrcrcr}
2x_1 & + & 4x_2 & + & 6x_3 & = & 6 \\
x_1 & + & 3x_2 & + & 4x_3 & = & 4 \\
 & & x_2 & + & ax_3 & = & 1
\end{array}
$$

1.1. (1 point) Find A and $\vec{b}$ such that the above system is in the form $A\vec{x} = \vec{b}$ (i.e. an matrix equation).

1.2. (5 points) Explicitly showing *all* the steps reduce the *augmented matrix* into reduce echelon form. (Hint: there are 2 cases for the rest of the problem.)

1.3. (1 point) What is the rank of A?

1.4. (1 point) Is there an a such that the system does not have a solution? Why or why not?

1.5. (2 points) Find the general solution (in vector form) for the above system.

2. (6 points) Let $\vec{x}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$, $\vec{x}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 4 \\ 0 \\ b \end{bmatrix}$.

2.1. Find a value of b such that $\vec{y}$ *is* a linear combination of $\vec{x}_1$ and $\vec{x}_2$.

2.2. Find a value of b such that $\vec{y}$ *is not* a linear combination of $\vec{x}_1$ and $\vec{x}_2$.

3. (4 points) Both of the following statements are false. Provide an example.

3.1. (2 points) If A is a 5×4 matrix and has rank 4 then for all $\vec{b} \in \mathbb{R}^5$, $A\vec{x} = \vec{b}$ is consistent.

3.2. (2 points) If A is an upper triangular matrix then the rank of A is the number of non-zero entries on A's diagonal.