Math 221 – Linear Algebra
Quiz 1

1. (This is based on problem 76 on page 90.) The color of light can be represented in a vector, $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$, where R is the amount of red light, G is the amount of green light and B is the amount of blue light. The human eye and the brain transforms the incoming signal into the signal $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$, where the intensity $I = \frac{R+G+B}{3}$, the long-wave signal $L = R - G$ and the short wave signal $S = B - \frac{R+G}{2}$.

1.1. Find the matrix $\mathcal{A}$ of the transformation taking $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ to $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$.

1.2. Find $\mathcal{A}^{-1}$.

1.3. Consider a pair of yellow sunglasses for water sports which cuts out all blue light but passes all red and green light. Find a 3×3 matrix $\mathcal{B}$ that represents the transformation incoming light undergoes as it passes through the sunglasses.

1.4. Is $\mathcal{B}$ invertible?

1.5. Find the matrix for the composite transformation that light undergoes as it first passes through the sunglasses and then the eye.

1.6. As you put on the sunglasses, the signal you receive (the I, L, and S) undergoes a transformation. Find the matrix $\mathcal{M}$ of this transformation. (There is nice picture in the book if this is not clear enough.)

2. Let B be a $m \times n$ matrix with $m \geq n$. Let A be a matrix such that BA is invertible. (Hint: look at problems 31-35 from section 2.4)

2.1. What is the size of A?

2.2. Is $T(\vec{x}) = B\vec{x}$ onto? Or equivalently, for all $\vec{b} \in \mathbb{R}^m$ there exists a vector $\vec{x} \in \mathbb{R}^n$ such that $B\vec{x} = \vec{b}$. Or equivalently, the linear system $B\vec{x} = \vec{b}$ is consistent for all vectors $\vec{b} \in \mathbb{R}^m$.

2.3. What is the rank of B?

2.4. What is the realtion between n and m?

2.5. Apply a lemma from the book (from class) to show that B and A are invertible.

3.

3.1. Find the matrix of the linear transformation $T_{\vec{a}} : \mathbb{R}^3 \to \mathbb{R}^3$ that projects each vector $\vec{x}$ onto the line given by $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

3.2. Find the matrix of the linear transformation $T_{\vec{b}} : \mathbb{R}^3 \to \mathbb{R}^3$ that projects each vector $\vec{x}$ onto the line given by $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

3.3. Now use the above informations to find the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that projects each vector $\vec{x}$ onto the plane $\mathcal{P}$ given by $\vec{a}$ and $\vec{b}$. (The plane $\mathcal{P}$ is the set of all linear combinations of $\vec{a}$ and $\vec{b}$.) (Hint: look at problem 40 from section 2.2)