

Student's name:.....

**Part 1. Multiple choice (5 points each)**

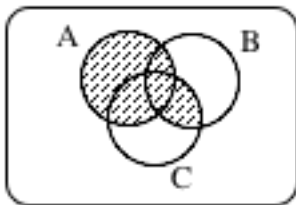
Below are five Venn diagrams. Which diagram represents the following set?

1.  $(A \cup B \cup C) - (A \cap B \cap C)$ .

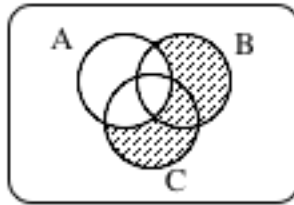
2.  $A' \cap (B \cup C)$ .

3.  $A' \cap B' \cap C'$ .

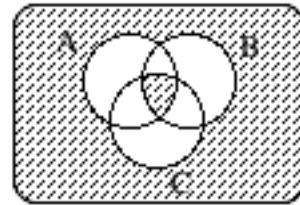
**Answers (to questions 1 to 4):**



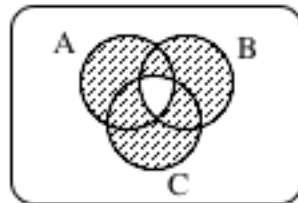
(a)



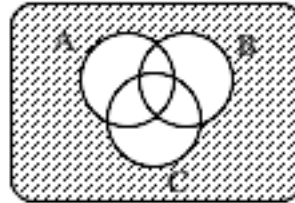
(b)



(c)



(d)



(e)

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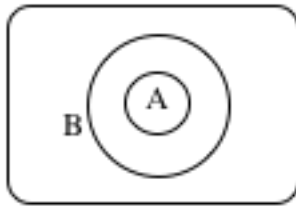
Below are five Venn diagrams. If the set  $C$  is not shown we are assuming  $C = \emptyset$ . Which diagram illustrates the following relation between sets?

4.  $A \cap B \cap C' = A \cap B$ .

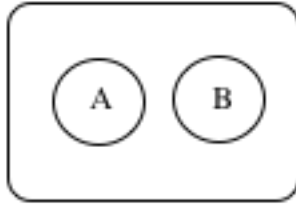
5.  $A \cap B \cap C = C$ .

6.  $B \cap C' \subseteq A$ .

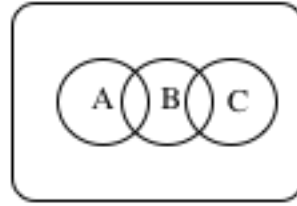
Answers (to questions 5 - 8):



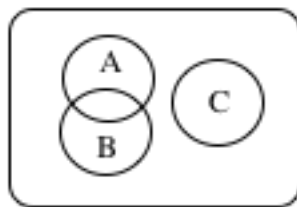
(a)



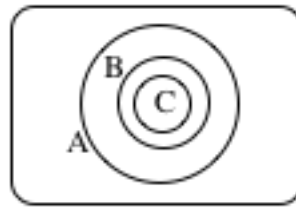
(b)



(c)



(d)



(e)

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7. Which of the following set theoretic equations is always satisfied?

**Answers** (to question 9):

- (a)  $(A \cap B)' \cup B = (A \cap B)' \cup A.$
- (b)  $(A \cup B)' \cap B = A \cap B.$
- (c)  $A \cup (B \cap A) = B.$
- (d)  $A \cap (B \cap A)' = \emptyset.$

**Part 2. Partial credit given**

8. Given mappings  $f: B \rightarrow C$  and  $g: A \rightarrow B$ , prove that if the composition  $f \circ g$  is surjective, then  $f$  is surjective.

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- 9 .The left column of the following table lists several binary operations on a set  $A$ , the remaining columns are headed by properties of binary operations. Fill in the table by placing "Y" in the proper column if the operation has the property and "N" if not and also writing in the identity element and the inverse if they exist.

Operation and set	Associative	Commutative	Has an identity element	Each element has an inverse
$A = \mathbf{Z}$ , Operation: Addition of integers				
$A = \mathbf{Z}$ , Operation: Multiplication of integers				
$A = \mathbf{Q}^+$ , Operation: Multiplication of rational numbers				
$A = \mathbf{R}^3$ , Operation: Cross product of vectors				
$A = \{\text{All } 2 \times 2 \text{ matrices over } \mathbf{R}\}$ , Operation: matrix multiplication				
$A = \{\text{Non singular } 2 \times 2 \text{ matrices over } \mathbf{R}\}$ , Operation: matrix multiplication				
$A = \{\alpha, \beta, \gamma\}$ , Operation $*$ given by the table below				

**Table for the Operation  $*$**

$*$	$\alpha$	$\beta$	$\gamma$
$\alpha$	$\alpha$	$\beta$	$\gamma$

$\beta$	$\beta$	$\gamma$	$\gamma$
$\gamma$	$\gamma$	$\alpha$	$\beta$

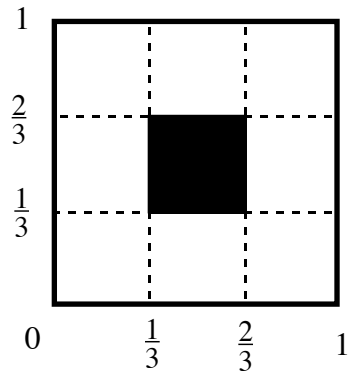
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10. The left column of the following table lists several binary relations on a set  $A$ , the remaining columns are headed by properties of binary relations. Fill in the table by placing "Y" in the proper column if the operation has the property and "N" if not

Relation and set	Reflexive	Symmetric	Transitive
$A = \mathbf{Z}$ , Relation: $x < y$			
$A = \mathbf{Z}$ , Relation: $x \geq y$			
$A = \mathbf{Q}$ , Relation: $x R y$ if and only if $y - x$ is an integer			
$A = \mathbf{R}^2$ , Relation: $x R y$ if and only if $x^2 = y^2$			
$A = \{\text{people}\}$ , Relation: $x R y$ if and only if $x$ is $y$ 's first cousin.			
$A = \mathbf{R}$ , Relation represented by the graph below.			

**Graph of the relation**

(The frame and the dotted lines do not belong to the graph)



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11. Show on an example that the distributive law  $x + (y z) = (x + y) (x + z)$  does not hold for addition and multiplication of integers.

12. Prove using mathematical induction that  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{1}{n+1}$ .

13. Find the greatest common divisor of the numbers 4625 and 444 and represent it in the form  $4625n + 444m$ ,

**Answer:**  $(4625, 444) = \dots\dots\dots$ ,  $n = \dots\dots\dots$ ,  $m = \dots\dots\dots$

**Work:**



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14. Show on an example of three integers that it is possible that  $n \mid ab$ , while neither  $n \mid a$  nor  $n \mid b$ .

Under what conditions will  $n \mid ab$  imply that  $n \mid a$ ?

15. Find all solutions of the congruence

$$21x \equiv 7 \pmod{14}.$$

If no solution exists, state so.