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Part 1. Multiple choice (5 points each)

1. Which of the following sets of 2×2 matrices is are subgroups of the group of all nonsingular 2×2 matrices over **R:**

$$A = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbf{Z},\ ad\text{-}bc = 1 \right\}$$

,

$$B = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbf{Z},\ ad\text{-}bc > 0 \right\},$$

$$C = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbf{R},\ ad\text{-}bc = 1 \right\},$$

$$D = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbf{R},\ ad\text{-}bc > 0 \right\}.$$

Answers:

(a) B only, (b) B and C only, (c) A and C only,

(d) A, C and D only, (e) all four .

2. Which of the following subgroups of $\boldsymbol{D}_4$ are normal subgroups?

$H = \{e, f\}$, $K = \{e, r, r^2, r^3\}$, $L = \{e, r^2\}$.

Answers:

(a) H only, (b) K only, (c) K and L only, (d) H and K only,
(e) all three

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3. Compute the following product

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}.$$

4. Find the conjugate of $(1\ \ 2)(3\ \ 4) \in S_4$ by $(1\ \ 2\ \ 3)$.

Answers (to questions 3 and 4):

(a) (1 2 3 4), (b) (1 3 2 4), (c) (1 2)(1 3),

(d) (1 3)(2 4), (e) (1 4)(2 3).

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5. How many nontrivial proper subgroups of the group $\mathbf{U}_{24}$ are there?

6. Find the order of the element $(1\ \ 2\ \ 3\ \ 4\ \ 5)$ of the group S_5 .

7. Find the index of the cyclic subgroup $\langle(1\ \ 2\ \ 3\ \ 4\ \ 5)\rangle$.

Answers (to questions 3 to 5):

(a) 5, (b) 7, (c) 12, (d) 24, (e) 30

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Answers (to questions 3 to 5):

(a) 5, (b) 7, (c) 12, (d) 24, (e) 30

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Part 2. Partial credit given

8. An example in the textbook showed that the subset $\{[2], [4], [6], [8]\} \subseteq \mathbf{Z}_{10}$ is a group under multiplication with identity element [6]. Generalize this result by proving that for every prime number p the subset $\{2k | k = 0, 1, ..., p - 1\} \subseteq \mathbf{Z}_{2k}$ is a group under multiplication mod $2p$ with identity element $p - 1$.

9. List all left cosets of the subgroup $\{e, (1\ 2)(3\ 4)\} < A_4$.

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10. The subset $\mathbf{Z}^+ \subseteq \mathbf{Z}$ is closed uder addition but is not a subgroup of $\mathbf{Z}$.
Prove that if G is a **finite** group, then every subset closed under the group operetion **is** a subgroup.

11. Let $f_a : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be given by $f_a(x, y) = (x + a,\ y + a)$. Prove that $\{f_n \mid n \in \mathbf{Z}\}$ with composition of mappings as operation is a group isomorphic to the group $\mathbf{Z}$.

12.

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13.

14.

15.