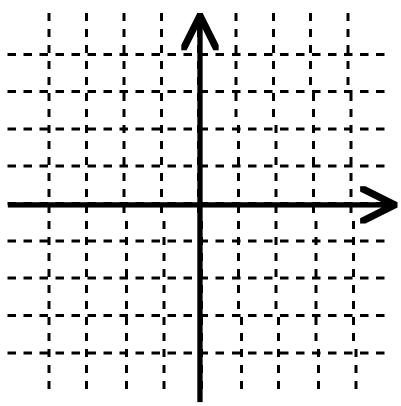
Student's name:...

1. Let $z = 3 + i$, $u = 4 - i$. Compute the following

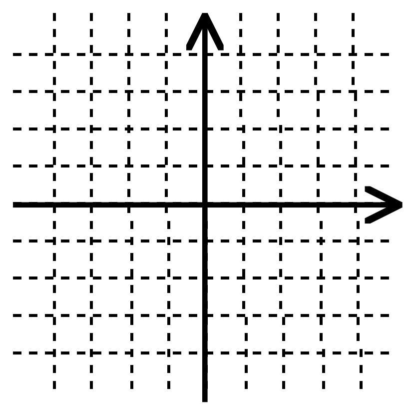
a) $\frac{z}{u} =$, b) $z\,u =$, c) $|z| =$.

2. Convert the number $2 - 2i$ to trigonometric form.

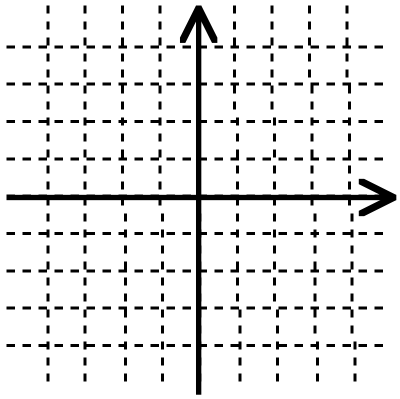
3. Graph the folowing equations

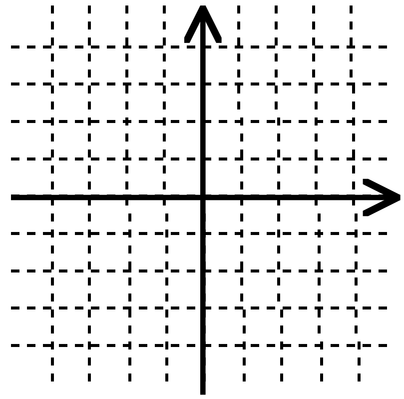


(a) $z + z = 4$



(b) $z - z = 4i$





c) $z \quad z = 9$

d) $z - z = 4$

4. Find <u>all</u> fourth roots of $-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$. Write them in the form $a + bi$.

5. Knowing that $1 - i\sqrt{3}$ is a cubic root of -8 and that $-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$. is a primitive root of unity find all cubic roots of -8 .

6. Prove that the set $\{z \in \mathbf{C} \mid |z| = 1\}$ forms a group under comlex multiplication. Is this subset a subring of the ring $\mathbf{C}$?

7. Let $R = \mathbf{Z} \oplus \mathbf{Z} = \{(x, y) \mid x, y \in \mathbf{Z}\}$ with addition and multiplication defined by

$$(x, y) + (u, v) = (x + u, y + v), \quad (x, y)(u, v) = (xv, yv)$$

and let $S = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \mid x, y \in \mathbf{Z} \right\}$ with matrix operations.

a) Prove that R and S are commutative rings with unity.

b) Are they isomorphic? If so, show the isomorphism; if not explain why.

c) Which of them, if any, is an integral domain? Prove your assertion.

8. Let $R = \left\{ \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \mid x, y \in \mathbf{Z} \right\}$ with matrix operations and let the mapping $\phi : R \to \mathbf{Z}$ be given by $\phi\left(\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \right) = x$.

a) Prove that R is a ring.

b) Is this ring commutative?

c) Is there a unity in R?

d) Is ϕ a ring homomorphism?

e) If yes, what is $\ker(\phi)$?

f) Is the subring $\left\{ \begin{pmatrix} 0 & y \\ 0 & 0 \end{pmatrix} \mid y \in \mathbf{Z} \right\}$ an ideal?

g) Is the subring $\left\{ \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} \mid x \in \mathbf{Z} \right\}$ an ideal?

Prove each of your answers.

9. The addition and multiplication tables on the set $S = \{0, e, a, c\}$ of four elements are as follows:

$+$	0	e	a	b
0	0	e	a	b
e	e	0	b	a
a	a	b	0	e
b	b	a	e	0

$\cdot$	0	e	a	b
0	0	0	0	0
e	0	e	a	b
a	0	a	b	e
b	0	b	e	a

Granted that the two operations satisfy the associative and distributive laws, check whether S with these operations is

a) a ring

b) an integral domain

c) a field

10. What is the characteristic of the following rings?

a) $\mathbf{Z}_4 \oplus \mathbf{Z}_6$ ....

b) $\mathbf{Z}_7$ ....

c) $\mathbf{Z}_9$ ....

d) $\mathbf{Z}_6$ ....

e) $\mathbf{Z}$ ....

11. In the following list of rings put a checkmark next to those that are integral domains.

❑ $\mathbf{Z}_6$, ❑ $\mathbf{Z}_7$, ❑ $\mathbf{R}$, ❑ $\mathbf{C}$, ❑ $\{a + bi \mid a,b \in \mathbf{Z}\}$, ❑ $\left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \mid x, y \text{ ▯ } \mathbf{Z}\right\}$

12. Let I and J be ideals of a commutative ring R. Prove that the set

$$K = \{x + y \mid x \in I, y \in J\}$$

is an ideal of R.