Student's name:...

1. Construct the multiplication table for the subgroup H of D_6 generated by the set $A = \{fr^2, fr^4\}$. What is the order of H? What is the index of H in D_6?

2. Find the conjugate of the transposition (2 3) by the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$.

3. Show that the conjugate of an odd permutation is an odd permutation and the conjugate of an even permutation is an even permutation.

4. What is the smallest order of a nonabelian group? Show an example of a nonabelian group of this order.

5. Does there exist an abelian group of order 1991? If yes give an example. If no say why?

6. Show that a group, which satisfies the identity $(xy)^2 = x^2 y^2$, is abelian.

7. Verify that $N = \{[1], [9]\}$ is a normal subgroup of $G = \boldsymbol{U}_{20}$. List all elements of the factor group G/N and construct the multiplication table for $G?N$.

8. Verify that the mapping $\phi : \boldsymbol{M} \to \mathbf{R}^*$, where $\boldsymbol{M}$ is the multiplicative group of all 2×2 non singular matrices and $\mathbf{R}^*$ is the multiplicative group of nonzero real numbers, given by $\phi(x) = \det x$ is a homomorphism. What is the kernel of this homomorphism?

9. Verify that the mapping $\psi : \mathbf{R}^* \to \mathbf{R}$ from the multiplicative group of nonzero real numbers into the additive group of all real numbers given by formula $\phi(x) = \ln |x|$ is a homomorphism. What is the kernel of this homomorphism?

10. Show that an epimorphism $\phi : G \rightarrow G'$ is an isomophism if and only if $\ker\phi = \{e\}$.

11. What are the possible orders of nontrivial subgroups of a group of order 36? Which of them are necessarily realized in every group of order 36?

12 True or false?

(a) Every abelian group is cyclic.

(b) Every cyclic group is abelian.

(c) Two cyclic groups of the same order are isomorphic.

(d) Any two groups of the same order are isomorphic.

(e) Any two abelian groups of the same order are isomorphic.

(f) A homomorphic image of an abelian group is abelian.

(g) If some homomorphic image of a group G is abelian, then G itself is abelian.

(h) If H is a subgroup of G and N is a normal subgroup of G, then $H \cap N$ is a normal subgroup of H.

(i) If a normal subgfroupo N of a group G is abelian and the factror group G/N is abelian, then G must be abelian.