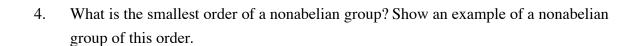
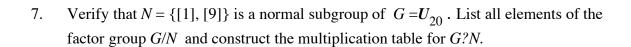
1. Construct the multiplication table for the subgroup H of  $D_6$  generated by the set  $A = \{fr^2, fr^4\}$ . What is the order of H? What is the index of H in  $D_6$ ?

2. Find the conjugate of the transposition (2 3) by the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ .

3. Show that the conjugate of an odd permutation is an odd permutation and the conjugate of an even permutation is an even permutation.



6. Show that a group, which satisfies the identity  $(xy)^2 = x^2 y^2$ , is abelian.



8. Verify that the mapping  $\phi : M \to \mathbb{R}^*$ , where M is the multiplicative group of all  $2 \times 2$  non singular matrices and  $\mathbb{R}^*$  is the multiplicative group of nonzero real numbers, given by  $\phi(x) = \det x$  is a homomorphism. What is the kernel of this homomorphism?

9. Verify that the mapping  $\psi : \mathbf{R}^* \to \mathbf{R}$  from the multiplicative group of nonzero real numbers into the additive group of all real numbers given by formula  $\phi(x) = \ln |x|$  is a homomorphism. What is the kernel of this homomorphism?

10.	Show that an epimorphism $\phi: G \to G'$ is an isomophism if and only if $\ker \phi = \{e\}$ .
11.	What are the possible orders of nontrivial subgroups of a group of order 36? Which of them are necessarily realized in every group of order 36?

- 12 True or false?
  - (a) Every abelian group is cyclic.
  - (b) Every cyclic group is abelian.
  - (c) Two cyclic groups of the same order are isomorphic.
  - (d) Any two groups of the same order are isomorphic.
  - (e) Any two abelian groups of the same order are isomorphic.
  - (f) A homomorphic image of an abelian group is abelian.
  - (g) If some homomorphic image of a group G is abelian, then G itself is abelian.
  - (h) If H is a subgroup of G and N is a normal subgroup of G, then  $H \cap N$  is a normal subgroup of H.
  - (i) If a normal subgroupo N of a group G is abelian and the factor group G/N is abelian, then G must be abelian.