MIDTERM MATH 436 PDE PART ONE

0. Read Sec 0.4 and 2.5 from the book

1. Find the Fourier series for the function f (x):

   a) $f(x) = 5x \quad , \quad 0 < x \le 2L \quad , \; f(x+2L) = f(x)$

   b) $f(x) = 5x \quad , \quad -L < x \le L \quad , \; f(x+2L) = f(x)$

2. Find the complex Fourier series for the function

$$f(x) = \begin{cases} 1 & 0 < x < L/2 \\ -1 & L/2 < x < L \end{cases} \quad , \quad f(x+L) = f(x)$$

3. Find the Fourier series for the function $f(x) = |\sin x|$ by using the fact that the function has period π. Give a formula for σ_n^2 (the mean square error).

4. A function $f(x)$ is said to be even and odd–harmonic if it satisfies the conditions

$$f(-x) = f(x) \quad , \quad f(L+x) = -f(L-x) \, .$$

Show that such a function is 4L–periodic.

5. Solve the following problems:

a) $$\frac{\partial u}{\partial t} - K \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for} \quad 0 < x < \pi \, , \quad t > 0$$

$$u(0,t) = u(\pi,t) = 0$$
$$u(x,0) = \sin^3 x$$

b) $$\frac{\partial u}{\partial t} - K \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for} \quad a < x < b, \quad t > 0$$

$$u(a,t) = 0$$
$$u(b,t) = 0$$
$$u(x,0) = (x-a)(b-a)$$

6. Consider the following problem:

$$y_{tt} = c^2 y_{ss} \qquad t > 0 \ , \quad 0 < s < L$$

$$y(0;t) = 0 = y(L;t)$$

$$y(s;0) = \begin{cases} \dfrac{3}{L} s \ , & 0 \le s \le \dfrac{L}{3} \\ -\dfrac{3}{2L} s + \dfrac{3}{2} \ , & \dfrac{L}{3} \le s \le L \end{cases}$$

$$y_t(s;0) = 1$$

Find the solution in the form of d'Alembert formula and graph it for $t = \dfrac{L}{2C}$ and $t = \dfrac{L}{4C}$.