MIDTERM MATH 436 PDE PART TWO

1. Find the Fourier series representation of the function:

a) $f(x) = \begin{cases} 2, & 0 < x < \pi \\ -1, & -\pi < x < 0 \\ \frac{1}{2}, & x = -\pi, 0, \pi \end{cases}$

$$f(x + 2\pi) = f(x)$$

2. Determine the even and odd periodic extensions and Fourier sine and cosine series for the function:

$$f(x) = \sin 2x \quad , \quad 0 < x < \pi$$

3. Find the solution of the following problem:

$$U_{tt} = C^2 U_{xx} - 2x$$
$$U(0,t) = U(\pi, t) = 0 \quad , \quad t > 0$$
$$U(x,0) = 0 \qquad 0 \le x \le \pi$$
$$U_t(x,0) = 1$$

4. What is Green's function for the heat equation?